21-268 Multidimensional Calculus: Midterm 2.

Wed 11/04

- *This is a closed book test. No calculators or computational aids are allowed.*
- *You have 50 minutes. The exam has a total of 5 questions and 47 points.*
- *You may use any result from class or homework* ***PROVIDED*** *it is* independent *of the problem you want to use the result in. (You must also* ***CLEARLY*** *state the result you are using.)*
- *The questions get progressively harder (though Q5(a) and (b) might be easier than Q4).*

[5] 1. Let $f(x, y) = \ln(x^2 + y^2)$. Compute $\partial_x \partial_y f$.

[5] 2. Find a vector normal to the surface $xy + ze^{z(x-y)} = 2$ at the point $(1, 1, 1)$.

[10] 3. Let $f(x, y) = 2x^2 + 2xy + 2x + 5y^2 - 8y$. Find all points at which $Df = 0$. Classify these points as local maxima, local minima or saddles.

[9] 4. Each of the figures below shows the level set $\{x \in \mathbb{R}^2 \mid f(x) = f(a)\}$ in a dashed line, and the level set $\{x \in \mathbb{R}^2 \mid g(x) = 0\}$ in a solid line. You may assume $f, g : \mathbb{R}^2 \to \mathbb{R}$ are two C^1 functions and $a = (0, 1) \in \mathbb{R}^2$. Further, in each case, assume $\partial_1 f(a) < 0$ and $\partial_2 f(a) > 0$.

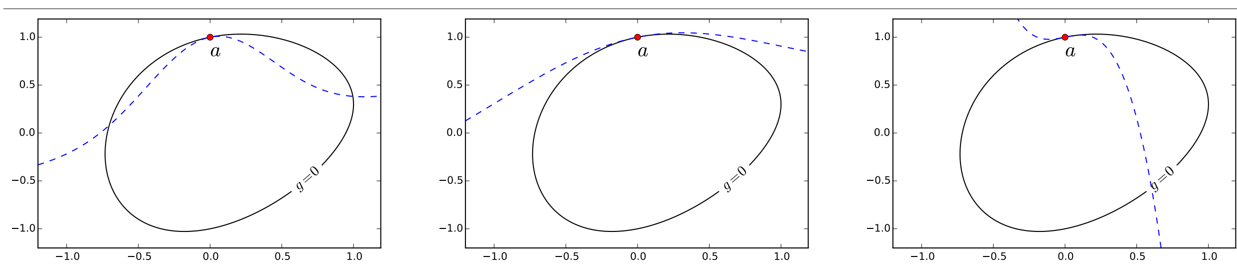
In each of the figures below, determine whether:

(a) f attains a constrained maximum at a, subject to the constraint $g = 0$,

(b) or f attains a constrained minimum at a, subject to the constraint $g = 0$,

(c) or f does not attain either a constrained maximum or minimum at a, subject to the constraint $g = 0$.

[A correct answer is worth full credit; a blank answer is worth 33% credit, and an incorrect answer is worth no credit.]



Answer: ______ Answer: ______ Answer: ______

5. Let $g : \mathbb{R}^2 \to \mathbb{R}$ be a C^2 function, and $\Gamma = \{(x_1, x_2) \mid g(x_1, x_2) = 0\}$. Assume

$$g(0,0) = 0, \quad \partial_1 g(0,0) = a, \quad \partial_2 g(0,0) = b, \quad \partial_1^2 g(0,0) = c, \quad \partial_1 \partial_2 g(0,0) = d, \quad \text{and} \quad \partial_2^2 g(0,0) = e.$$

[2] (a) State a condition that will guarantee the existence of a differentiable function f such that

$$f(0) = 0 \quad \text{and} \quad g(x, f(x)) = 0$$

for all $x \in \mathbb{R}$ in a sufficiently small neighbourhood of 0. [Your condition should only involve a, b, c, d and e. No proof or explanation is required.]

[6] (b) Assuming the existence of the function f above, compute $f'(0)$ in terms of a, b, c, d and e.

[10] (c) Assuming that the function f above is C^2, compute $f''(0)$ in terms of a, b, c, d and e.