Abstracts of talks for (Your Regular) Workshop on Extremal Graphs and Hypergraphs

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Induced subgraphs of Ramsey graphs with many distinct degrees

Boris Bukh

Joint work with: Benny Sudakov

An induced subgraph is called *homogeneous* if it is either a clique or an independent set. Let $\hom(G)$ denote the size of the largest homogeneous subgraph of a graph G. The classic result in Ramsey theory asserts that $\hom(G) \geq \frac{1}{2}\log n$ for any graph G on n vertices, and it is best possible apart from the value of the multiplicative constant. We study properties of graphs on n vertices with $\hom(G) \leq C \log n$ for some constant C. We show that every such graph contains an induced subgraph of order αn in which $\beta\sqrt{n}$ vertices have different degrees, where α and β depend only on C. This proves a conjecture of Erdős, Faudree and Sós.

Induced Ramsey-type theorems

Jacob Fox

We present a unified approach to proving Ramsey-type theorems for graphs with a forbidden induced subgraph which can be used to extend and improve the earlier results of Rödl, Łuczak-Rödl, Prömel-Rödl, Erdős-Hajnal, and Nikiforov. The proofs are based on a simple lemma (generalizing one by Graham, Rödl, and Ruciński) that can be used as a replacement for Szemerédi's regularity lemma, thereby giving much better bounds. The same approach can be also used to show that pseudo-random graphs have strong induced Ramsey properties. This leads to explicit constructions for upper bounds on various induced Ramsey numbers. This talk is based on joint work with Benny Sudakov.

A proof of the stability of extremal graphs.

Zoltan Furedi

We present a concise, contemporary proof (i.e., one using Szemerédi's regularity lemma) for the following classical stability result of Simonovits 1968:

If an n-vertex F-free graph G is almost extremal, $\mathrm{chr}(F) = p + 1$, then the structure of G is close to a p-partite Turán graph. More precisely, for every graph F and $\varepsilon > 0$ there exists a $\delta > 0$ and a bound n_0 (depending on F and ε) such that if $n > n_0$ and

$$e(G) > (1 - \frac{1}{p})\binom{n}{2} - \delta n^2$$

then one can change (add and delete) at most εn^2 edges of G and obtain a complete p-partite graph.

Regularity and Ramsey problems for hypergraphs

Penny Haxell

Cyclic Arrangements of k-sets with Local Intersection Constraints

Tao Jiang

Let n, k be positive integers with $k < n$. Let $\binom{[n]}{k}$ denote the family of all k-subsets of $\{1, 2, \ldots, n\}$. A cyclic ordering of (all the members of) $\binom{[n]}{k}$ is *s-overlapping* if every s consecutive members in this ordering are pairwise intersecting. Let $f(n, k)$ denote the maximum s such that there exists an s-overlapping ordering of $\binom{[n]}{k}$.

For each $n \geq 6$ we show that $f(n, 2) = 3$. We show that $f(n, 3)$ equals either $2n - 8$ or $2n - 7$ when n is sufficiently large, conjecturing that $2n - 8$ is the correct value. For each $k \geq 4$ and n sufficiently large we show that

$$\frac{2n^{k-2}}{(k-2)!} - \frac{(\frac{7}{2}k - 2)n^{k-3}}{(k-3)!} + O(n^{k-4}) \leq f(n,k) \leq \frac{2n^{k-2}}{(k-2)!} - \frac{(\frac{7}{2}k - c)n^{k-3}}{(k-3)!} ,$$

where c is an absolute constant. This is joint work with Manley Perkel and Dan Pritikin.

Constrained Ramsey numbers

Po-Shen Loh

For two graphs S and T, the constrained Ramsey number $f(S, T)$ is the minimum n such that every edge coloring of the complete graph on n vertices (with any number of colors) has a monochromatic subgraph isomorphic to S or a rainbow subgraph isomorphic to T. Here, a subgraph is said to be rainbow if all of its edges have different colors. It is an immediate consequence of the Erdős-Rado Canonical Ramsey Theorem that $f(S, T)$ exists if and only if S

is a star or T is acyclic. Much work has been done to determine the rate of growth of $f(S,T)$ for various types of parameters. When S and T are both trees having s and t edges respectively, Jamison, Jiang, and Ling showed that $f(S,T) \leq O(st^2)$ and conjectured that it is always at most $O(st)$. They also mentioned that one of the most interesting open special cases is when T is a path. We study this case and show that $f(S,P_t) = O(st\log t)$, which differs only by a logarithmic factor from the conjecture. This substantially improves the previous bounds for most values of s and t.

Joint work with Benny Sudakov.

On the edit distance in graphs

Ryan Martin

A hereditary property of graphs is one that is closed under induced subgraphs. For example, the absence of an **induced** copy of a 4-cycle is a hereditary property. The edit distance of a graph G from a hereditary property is the fewest number of edge-deletions or edge-insertions required to transform G into a graph G' that satisfies the hereditary property.

For a fixed hereditary property $\mathcal{H}$, we study the maximum edit distance from $\mathcal{H}$ over all n-vertex graphs. We summarize known results for this invariant and provide a new technique for finding the asymptotic value of this quantity which has produced new results.

This talk surveys joint work with Maria Axenovich, André Kézdy and Jozsef Balogh.

An alteration principle for bounding the chromatic numbers of some geometric graphs

A.M. Raigorodskii

In this talk, we shall present a new approach for finding non-trivial lower bounds on the chromatic numbers of distance graphs and graphs of diameters. Some related questions of combinatorial geometry (such as Nelson – Hadwiger and Borsuk problems) and Ramsey theory will be discussed.

Density theorems for bipartite graphs

Benny Sudakov

In this talk, we present several Turan-type theorems which show how to find a copy of a sparse bipartite graph H in a graph G of positive density. Our results imply several new bounds on Ramsey numbers of bipartite graphs and improve and generalize earlier results of various researchers. The proofs combine probabilistic techniques with some combinatorial ideas. These same techniques can be modified to give new bounds on multicolor induced Ramsey numbers and new results related to the Erdos-Hajnal problem on the largest homogeneous set in graphs with a forbidden induced subgraph.

This is joint work with Jacob Fox.

Limits of hypergraphs and the hypergraph regularity lemma. A non-standard approach

Balzs Szegedy

We develop an analytic language using ultra product of finite measure spaces. In this language, the famous Hypergraph Regularity Lemma and Hypergraph Removal Lemma (by Rodl-Skokan-Nagle-Schacht, Gowers, Tao) are simple consequences of classical theorems of real analysis such as Lebesgue's Density Theorem, Radon-Nykodym theorem, etc... A delicate fact is that our proof of the Hypergraph Removal Lemma follows immediatley from the Lebesgue Density Theorem without using any regularity result. We prove the existence of a limit object for dense hypergraph sequences which is a generalization of the limit object by Lovasz and Szegedy for graphs.

Joint work with: Gabor Elek

Some results about embedding bounded degree (hyper-) graphs in dense (hyper-) graphs

Endre Szemerédi

We are going to prove

1. If a minimum degree of a graph G ($|V(G)| = n$) is $\geq \frac{n}{2} + \log n$ then it contains any bounded degree tree T ($|V(T)| = n$).
2. If the minimum degree of G ($|V(G)| = n$) is at least $\frac{2n}{3}$ then G contains a square of a Hamilton cycle.
3. If in a 3-uniform hypergraph H ($|V(H)| = n$) the codegree of every pair $\{x, y\}$, $x, y \in V(H)$ is at least $\frac{n}{2}$, then H contains a HAmilton cycle. We can prove these results with or without using regularity lemmas.

Nearly optimal embedding of trees

Jan Vondrak

We present a simple randomized algorithm which finds nearly optimal embeddings of large trees in certain classes of graphs. The size of the tree T can be a constant fraction of the size of the graph G, and the maximum degree of T can be close to the minimum degree of G. For example, we prove that any graph of minimum degree d without 4-cycles contains every tree of size ϵd^2 and maximum degree at most $(1-2\epsilon)d$. As there exist d-regular graphs without 4-cycles of size $O(d^2)$, this result is optimal up to constant factors. We obtain similar results for graphs of given girth, graphs with no complete bipartite subgraph $K_{s,t}$, random graphs and certain pseudorandom graphs.

Joint work with Benny Sudakov.