Syntax! Semantics! Databases?

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We begin with a basic question: when is an infinite set of formulas in a free variable x over an infinite set of parameters A simultaneously satisfied in a particular model M containing A ? Does the situation change if we expand the language by adding a predicate naming A? We discuss this notion in an elementary way. Then, we sketch connections with other problems in model theory and discuss at more length the area that attracted to me this issue: Embedded finite model theory provides a setting for the discussion of databases with arithmetic operations (generalizing the finite set of tuples interpretation). The methods discussed here provide 'generic collapse' results in embedded finite model theory.

The following recent result (joint with Bektur Baizhanov) illuminates the situation. A subset A of a structure M is *benign* if for any c, $\mathrm{tp}(\mathrm{c}, \mathrm{A})$ implies $\mathrm{tp}_*(\mathrm{c}, \mathrm{A})$ (where $*$-types are in the language with a predicate naming A) (i.e. the second question above has a negative answer for A). Theorem: If A is a uniformly weakly benign subset of a stable structure and the 'induced' structure on A is stable then T^* is stable. Further work with Shelah has shown that at least in superstable theories there are many weakly benign subsets.