Calculus I, 21-111
Test 3
April 30

Name:

Note: No calculators or electronic devices of any sort are allowed. Show all work necessary to obtain your answers. No credit will be given for answers without justification.

1. (20 points) Let $f(x) = e^{-3x}, g(x) = \ln(e^x + x^3), h(x) = xe^{\sqrt{x}}, k(x) = \frac{1}{\ln(x)}$. Give a simplified expression for each of the following:

   (a) $f'(0)$ $f'(x) = -3e^{-3x}$

   $f'(0) = -3e^0 = \boxed{-3}$

   (b) $g'(1)$ $g'(x) = \frac{1}{e^x + x^3}(e^x + 3x^2)$

   $g'(1) = \frac{1}{e + 1^3}(e^1 + 3 \cdot 1^2) = \boxed{\frac{3+e}{1+e}}$

   (c) $h'(4)$ $h'(x) = x \cdot e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2} + e^{\sqrt{x}}$

   $= \frac{1}{2}x^{1/2}e^{\sqrt{x}} + e^{\sqrt{x}}$

   $h'(4) = \frac{1}{2} \cdot 4^{1/2} e^{\sqrt{4}} + e^{\sqrt{4}} = \frac{1}{2} \cdot 2 \cdot e^2 + e^2 = e^2 + e^2 = \boxed{2e^2}$

   (d) $k'(e)$ $k'(x) = -\frac{1}{\ln(x)^2} \cdot \frac{1}{x} = \frac{-1}{x(\ln(x))^2}$

   $k'(e) = \frac{-1}{e(\ln(e))^2} = \frac{-1}{e \cdot 1^2} = \boxed{\frac{-1}{e}}$

2. (20 po nts) Solve each of the following equations for x:

(a) $3e^{x/2} - 12 = 0$

$3e^{x/2} = 12$

$e^{x/2} = 4$

$\frac{x}{2} = \ln(4)$

$\boxed{x = 2\ln(4)}$

(b) $\ln(e \cdot x) + \ln(x^2) = 4$

$\ln(e) + \ln(x) + \ln(x^2) = 4$

$1 + \ln(x) + 2\ln(x) = 4$

$3\ln(x) = 3$

$\ln(x) = 1$

$x = e^1 = \boxed{e}$

(c) $4e^{x}e^{-2x} = 6$

$4e^{-x} = 6$

$e^{-x} = \frac{6}{4} = \frac{3}{2}$

$-x = \ln\left(\frac{3}{2}\right)$

$\boxed{x = -\ln\left(\frac{3}{2}\right) = \ln\left(\frac{2}{3}\right)}$

(d) $\ln((x^2+1)^3) = 9$

$3\ln(x^2+1) = 9$

$\ln(x^2+1) = 3$

$x^2 + 1 = e^3$

$x^2 = e^3 - 1$

$\boxed{x = \sqrt{e^3-1}}$

3. (10 p›ints) Sketch the curve $y = \ln(x) - x$ for values of $x > 0$, labeling clearl› all maximums, minimums, and points of inflection.

$y = \ln x - x$

$y' = \frac{1}{x} - 1 = 0$ if $\frac{1}{x} = 1$

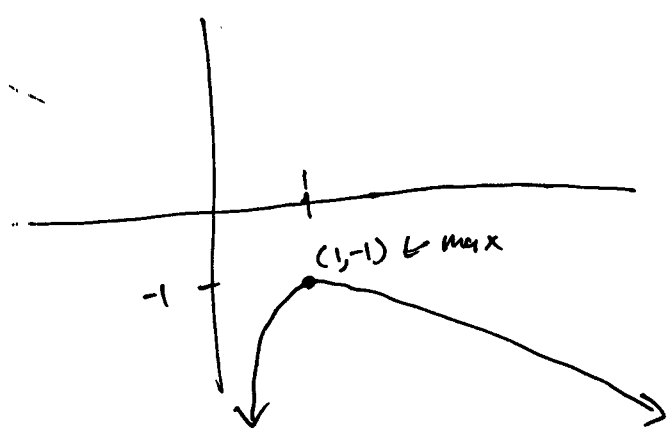
$x = 1$ ← critical pt. is $(1,-1)$

$y(1) = \ln(1) - 1 = 0 - 1 = -1$

$y'' = -\frac{1}{x^2} < 0$ for all x

so graph is concave down everywhere

$\Rightarrow (1,-1)$ is an abs. max, and no inflection pts



4. (20 points) A radioactive isotope decays with an exponential rate constant of $-\ln(2)$. At time $t = 0$, there are 10 grams of the isotope present. Let $y(t)$ represent the mass of the isotope present after t years.

(a) Give a simplified formula for $y(t)$.

(b) What is the half-life of this isotope?

(c) When will there be $\frac{5}{8}$ of a gram left?

a) $y(t) = 10e^{-\ln(2)t} = 10(e^{\ln(2)})^{-t}$
$= 10 \cdot 2^{-t}$

b) $y(0) = 10$

$1 \cdot 2^{-t} = 5$

$2^{-t} = \frac{5}{10} = \frac{1}{2}$

$1 = \frac{2^t}{2}$

$2 = 2^t$

$1 = t$

Since the mass goes from 10 to 5 in 1 yr, the half-life is $\boxed{1 \text{ yr}}$

c) $10 \cdot 2^{-t} = \frac{5}{8}$

$2^{-t} = \frac{1}{16}$

$\boxed{t = 4}$ after 4 yrs.

5. (15 points) How much must you invest in a savings account today to have a \$1000 balance at the end of two years if interest is given at a yearly rate of 12%?

   (a) compounded monthly?

   (b) compounded continuously?

$y =$ quantity invested today

a) $y\left(1+\frac{.12}{12}\right)^{12t} = y(1.01)^{12t}$

after 2 yrs ... $y(1.01)^{24} = 1000$

$$\boxed{y = \frac{1000}{(1.01)^{24}}}$$

b) $ye^{.2t} = 1000$

$t=2$: $ye^{.4} = 1000$

$$y = \frac{1000}{e^{.24}}$$

6. (15 pc ints) You take out a loan of \$50,000, with 6% yearly interest, compounc ed monthly.

(a) I you wish to pay off the loan in 10 years, what must your monthly payment be?

(b) I you pay \$400 per month, how long would it take you to pay off the l an?

monthly interest: $\frac{.06}{12} = .005$

a) 10 years = 120 months.

$$M = \frac{.005(50,000)}{1-(1.005)^{-120}}$$

b) $M = 1100$, $n = ?$

$$400 = \frac{.005(50,000)}{1-(1.005)^{-n}}$$

$$1-(1.005)^{-n} = \frac{250}{400} = \frac{5}{8}$$

$$+(1.005)^{-n} = \frac{5}{8} - 1 = +\frac{3}{8}$$

$$-n(\ln(1.005)) = \ln(\tfrac{3}{8})$$

$$-n = \frac{\ln(\frac{3}{8})}{\ln(1.005)}$$

$$n = -\frac{\ln(\frac{3}{8})}{\ln(1.005)}$$