Calculus I, 21-111
Test 1
February 16

Name

Recitation (circle yours): Klobusicky (TR 2:30) Mogin (TR 3:30)

Note: No calculators or any electronic devices of any sort are allowed. Show all work necessary to obtain your answers. No credit will be given for answers without justification.

1. (20 points) Let $f(x) = 7x^2 + 5x - 3, g(x) = 2/x, h(x) = 7, j(x) = 5\sqrt{x}$. Give a simplified expression for each of the following:

(a) $f''(x)$

$f'(x) = 14x + 5$

$f''(x) = 14$

(b) $h'(6)$ $h'(x) = 0$, so $h'(6) = 0$

(c) $f(g(x)) + h(x)$

$f(g(x)) = \frac{7\cdot 2^2}{x^2} + \frac{5\cdot 2}{x} - 3 = \frac{28}{x^2} + \frac{10x}{x^2} - \frac{3x^2}{x^2}$

$f(g(x)) + h(x) = \frac{28}{x^2} + \frac{10x}{x^2} - \frac{3x^2}{x^2} + \frac{7x^2}{x^2} = \frac{28 + 10x + 4x^2}{x^2}$

(d) $j'(25)$

$j'(x) = \frac{5}{2}x^{-1/2}$

$j'(25) = \frac{5}{2}(25)^{-1/2} = \frac{5}{2}\cdot\frac{1}{5} = \frac{1}{2}$

(e) $g''(2)$ $g' = \frac{-2}{x^2}$ $g''(x) = \frac{4}{x^3}$ $g''(2) = \frac{4}{2^3} = \frac{1}{2}$

2. (1. points) Find the derivative of the function $f(x) = x^2 - 4x + 1$ directly fr. n the definition.

$$f'(x) = \lim_{h \to .} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) + 1 - (x^2 - 4x + 1)}{h}$$

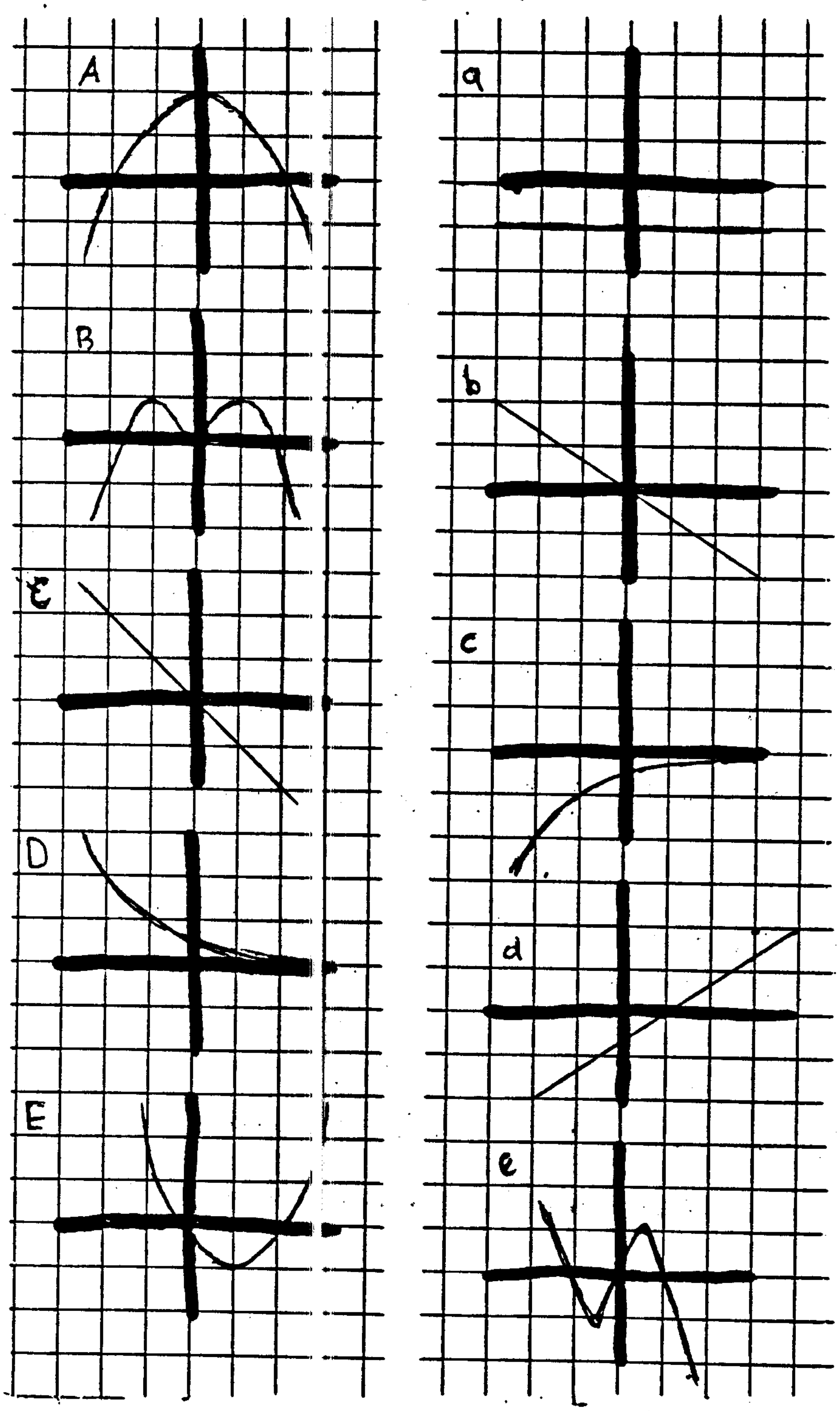
$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h + \cancel{1} - \cancel{x^2} + \cancel{4x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 4)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h - 4 = 2x - 4$$

3. (5 points) Match the graphs of the five functions A-E given below with their derivatives a-e. Explain your reasoning for one of the matches. You need not explain every one.



A ↔ b

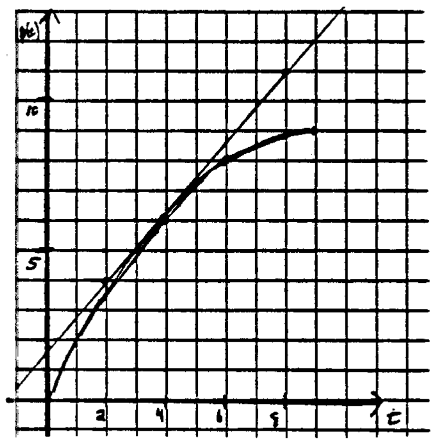
B ↔ e

C ↔ a

D ↔ c

E ↔ d

4. 15 points) Curtis runs along a straight path. His distance traveled (in meters) as a function of time (in seconds) $p(t)$ is graphed below, along with the line tangent to the graph at $t = 4$.



(a) How far has he gone after 6 seconds?

8 m

(b) What is his average velocity over the first four seconds?

$$\frac{p(4)-p(0)}{4} = \frac{6-0}{4} = \frac{3}{2} \text{ m/s}$$

(c) What is his instantaneous velocity after 4 seconds?

$\frac{11-4}{8-2} = \frac{7}{6}$, so slope of tan. line $= \frac{7}{6}$ m/s (Drawing poor, so any answer showing this idea gets credit)

(d) Is he going faster when $t = 2$ or when $t = 5$? Why?

He's going faster when $t=2$ since the slope is steeper there.

(e) Is $p''(3)$ positive or negative? Why?

$p''(3) < 0$ since the slope is decreasing at $t=3$ (the graph is concave down there)

5. 15 points) Let $f(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x - 1$. Find all points of the graph of ' at which the slope is -1. Give the equation for the tangent line at one f the points in $y = mx + b$ form.

$f'(x) = 2x^2 + x - 1$

To find where slope $= -1$, set $f' = -1$:

$2x^2 + x - 1 = -1$

$2x^2 + x = 0$

$x(2x+1) = 0 \Rightarrow f' = -1$ if $x = 0$ or $x = -\frac{1}{2}$

$f(0) = -1$

$f(-\frac{1}{2}) = \frac{2}{3}(-\frac{1}{2})^3 + \frac{1}{2}(-\frac{1}{2})^2 - (-\frac{1}{2}) - 1$

$= \frac{2}{3}(-\frac{1}{8}) + \frac{1}{2}(\frac{1}{4}) + \frac{1}{2} - 1$

$= -\frac{2}{24} + \frac{1}{8} + \frac{1}{2} - 1$

$= \frac{-2}{24} + \frac{3}{24} + \frac{12}{24} - \frac{24}{24} = \frac{-11}{24}$

So, pt. at which slope $= -1$ are $(0,-1)$, $(-\frac{1}{2}, -\frac{11}{24})$

Tan. line through $(0,-1)$ has equation

$y - (-1) = -1(x - 0)$

$y + 1 = -x$

$y = -x - 1$

6. (10 points) Let $f(x) = x^3 - 6x^2 + 9x + 5$. Sketch its graph. Label all maximums, minimums, and points of inflection.

$f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x-3)(x-1) = 0$ if $x = 1$ or 3

$f(1) = 1^3 - 6\cdot 1^2 + 9\cdot 1 + 5 = 9$

$f(3) = 3^3 - 6\cdot 3^2 + 9\cdot 3 + 5 = 5$

Crit. pts: $(1,9)$, $(3,5)$

		1		3	
$(x-3)$	$-$		$-$		$+$
$(x-1)$	$-$		$+$		$+$
f'	$+$	max	$-$	min	$+$

$f''(x) = 6x - 12 = 0$ if $x = 2$, $f(2) = 2^3 - 6\cdot 2^2 + 9\cdot 2 + 5$
$= 7$

		2	
f''	$-$		$+$

Put it together:

		1		2		3	
f'	$+$		$-$				$+$
f''	$-$		$-$				$+$

f