Review 3 Solutions

1a) $\left(e^{3x} e^{-x}\right)^2 = e^6$

$$\left(e^{2x}\right)^2 = e^6$$

$$e^{4x} = e^6$$

$$4x = 6$$

$$\boxed{x = \frac{6}{4} = \frac{3}{2}}$$

b) $\ln(3x+1) = 4$

$$3x+1 = e^4$$

$$3x = e^4 - 6$$

$$\boxed{x = \frac{e^4 - 6}{3}}$$

c) $\ln 2\, e^{-.3x} = 1$

$$e^{-.3x} = \frac{1}{\ln 2}$$

$$-.3x = \ln\left(\frac{1}{\ln 2}\right) = -\ln(\ln(2))$$

$$\boxed{x = \frac{\ln(\ln(2))}{.3}}$$

2 a) $y' = (x^2 \ln(x))\left(2x\ln(x) + x^2 \cdot \frac{1}{x}\right)$

$= x^2 \ln(x)\left(2x\ln(x) + x\right)$

$= x^3(\ln(x))^2 + 2x^3\ln(x)$

b) $y' = (6x^2+4)e^{4x} + (2x^3+4x) \cdot 4e^{4x}$

c) $y' = \frac{1}{1+e^x} \cdot e^x = \frac{e^x}{1+e^x}$

d) $y' = e^{\frac{-x^2}{2}} \cdot (-x)$

$= -xe^{-\frac{x^2}{2}}$

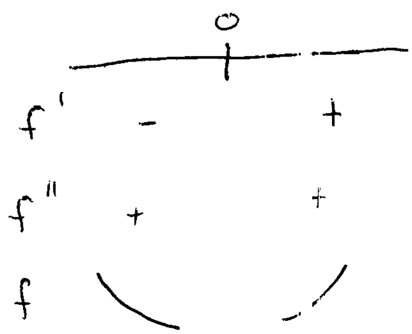
3) $f(x) = e^{x} - x$

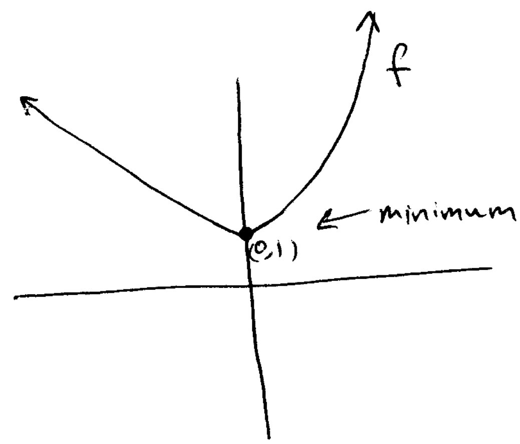
$f'(x) = e^{x} - 1 = 0 \quad$ if $e^{x} = 1$

$x = \ln(1) = 0$

$f''(x) = e^{x} > 0$ for all values of x

$f(0) = e^{0} - 0 = 1 - 0 = 1$





4) $y = 3x e^{x}$

$y' = 3e^{x} + 3x e^{x}$

$y'(0) = 3e^{0} + 3 \cdot 0 \cdot e^{0}$

$= 3e^{0} = 3$

$y(0) = 3 \cdot 0 e^{0} = 0$

tan. line has slope 3 & passes through point (0, 0), so equation of line is:

$y - 0 = 3(x - 0)$

$\boxed{y = 3x}$

5) a) $y = Ce^{rt}$ $\qquad$ $1990 \leftrightarrow t=0$

$$y(0) = Ce^{r\cdot 0} = 17,000,000$$

$$C = 17,000,000 = 17\cdot 10^6$$

$$y(7) = 17\cdot\cancel{10^6}\,e^{r\cdot 7} = 19.3\cdot\cancel{10^6}$$

$$e^{7r} = \frac{19.3}{17} \approx 1.135$$

$$7r = \ln(1.135) \approx 0.12689$$

$$r \approx \frac{.12689}{7} \approx 0.0181$$

$y(t)$ = population t yrs. after 1990

$$\boxed{= 17\cdot 10^6 e^{.018t}}$$

b) Jan. 1 2015 : $t = 15$

$$y = 17\cdot 10^6 e^{(.018)(15)}$$

$$\boxed{\approx 22.3\cdot 10^6}$$

c) $17\cdot\cancel{10^6}\,e^{.018t} = 30\cdot\cancel{10^6}$

$$e^{.018t} = \frac{30}{17}$$

$$.018t = \ln\left(\frac{30}{17}\right)$$

$$t = \frac{1}{.18}\ln\left(\frac{30}{17}\right) \approx 31.3$$

31 years after 1990 is $\boxed{2021}$

d) $y' = .018 \; 17\cdot 10^6 e^{.018t}$

$y'(31.3) = 018\cdot 17\cdot 10^6 e^{.018\cdot 31.3}$

$\approx \boxed{540{,}000 \text{ people per year}}$

6) a) $y(t) =$ radioactive grams after t years

$= Ce^{rt}$

$y(5.3) = Ce^{r(5.3)} = \frac{1}{2}y(0) = \frac{1}{2}C$

$\cancel{C}e^{5.3r} = \frac{\cancel{C}}{2}$

$e^{5.3r} = \frac{1}{2}$

$5.3r = \ln(\frac{1}{2}) = -\ln 2$

$r = \frac{-\ln 2}{5.3} \approx -0.131$

$y(0) = C = 15$

$y(t) = 15e^{-.131t}$

$y(10) = 15e^{-.131\cdot 10} \approx \boxed{4.06 \text{ grams}}$

b) $15e^{-.131t} = 1$

$e^{-.131t} = 1/15$

$\cancel{-}.131t = \ln\left(\frac{1}{15}\right) = \cancel{-}\ln(15)$

$t = \dfrac{\ln(15)}{.131} \approx \boxed{20.7 \text{ years}}$

7) a) $5000\left(1+\frac{.06}{12}\right)^{12\cdot5} \approx \boxed{\$6744.25}$

b) $5000e^{.06\cdot5} \approx \boxed{\$6749.29}$

c) $y'(t) = .06\cdot5000e^{.06t}$

$y'(0) = .06\cdot5000e^{.06\cdot0}$

$= .06\cdot5000 = 300$ dollars/yr

$y'(5) = .06\cdot5000e^{.06\cdot5} \approx 405$ dollars/yr

8) $P = 100{,}000$ (principal)

$r = \frac{.07}{12}$ (monthly interest)

$n = 20\cdot12 = 240$ months

$$M = \frac{\frac{.07}{12}\cdot100{,}000}{1-\left(1+\frac{.07}{12}\right)^{-240}} \approx \boxed{\$775.30}$$

9) $4000\left(\frac{(1.05)^{26}-1}{1.05-1}\right) \approx \boxed{204{,}453.82 \text{ dollars}}$

10) y = present value

$$y(1.04)^{10} = 15{,}000$$

$$y = \frac{15{,}000}{(1.04)^{10}} \approx \boxed{\$10{,}133.46}$$

11) x = intensity of sound at 108 db:

$$108 = 10 \log_{10}\left(\frac{x}{10^{-12}}\right)$$

$$10.8 = \log_{10}\left(\frac{x}{10^{-12}}\right)$$

$$10^{10.8} = \frac{x}{10^{-12}}$$

$$x = 10^{12} \cdot 10^{10.8} = 10^{22.8}$$

y = intensity at 100 db:

$$100 = 10 \log_{10}\left(\frac{y}{10^{-12}}\right)$$

$$\therefore\ y = 10^{10} \cdot 10^{12} = 10^{22}$$

$$\frac{x}{y} = \frac{10^{22.8}}{10^{22}} = 10^{.8} \approx 6.3$$

A 108 db sound is about 6.3 times more intense then a 100 db sound