

Calculus I, 21-111
Review problems for the third test
April 25

1. Solve each of the following equations for x :

(a) $(e^{3x} \cdot e^{-x})^2 = e^6$
(b) $\ln(3x + 6) = 4$
(c) $\ln(2)e^{-.3x} = 1$

2. Differentiate each of the following:

(a) $y = (x^2 \ln(x))^2$.
(b) $y = (2x^3 + 4x)e^{4x}$.
(c) $y = \ln(1 + e^x)$
(d) $y = e^{-x^2/2}$

3. Sketch a graph of the function $f(x) = e^x - x$, labeling all maximums, minimums, and points of inflection.
4. Find the equation of the tangent line to the graph of $y = 3xe^x$ at $x = 0$.
5. From January 1, 1990 to January 1, 1997, the population of Texas grew from 17 million to 19.3 million.
- (a) Assuming exponential growth, give a formula for population t years after 1990.
- (b) According to this model, how large would the population be on Jan.1, 2005? (It was estimated to be 22,859,968 by the census bureau.)
- (c) According to the model, in what year will the population reach 30 million?
- (d) What will the growth rate of the population be at that time?
6. The half-life of cobalt-60 is 5.3 years. Suppose there is 15 grams of it.
- (a) How much will still be radioactive in 10 years?
- (b) How long will it take until less than 1 gram is radioactive?
7. Suppose you invest \$5000 in a savings account.
- (a) If the account gives 6% annual interest, compounded monthly, how much will the account be worth in 5 years?
- (b) Suppose the 6% annual interest is compounded continuously. How much would the account be worth in 5 years?
- (c) In the continuous growth scenario, how fast is your money growing at time 0? At time 5 years?

8. Suppose you borrow \$100,000 to buy a house. The bank charges you 7% yearly interest, compounded monthly. If you plan to pay off the loan in 20 years, what will your monthly payment be?
9. Suppose you put \$4000 into a retirement savings account every year. The account pays 5% yearly interest. How much money will you have after 25 years?
10. Suppose someone offers to pay you \$15,000 in 10 years. Assuming you can safely invest money with a 4% yearly rate of return, what is the present value of their offer?
11. The decibel scale measures the loudness of sound according to the formula

$$\text{loudness} = 10 \log_{10} \left(\frac{x}{I_0} \right)$$

where x is the intensity of the sound measured in watts per square meter, and $I_0 = 10^{-12}$ watts per square meter is the least intense sound that a human ear can detect. How many times louder is a 108-decibel sound than a 100-decibel sound?