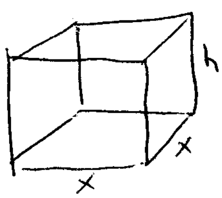
Calculus I, 21-111
Test 2 Make-up problems
~~March 30~~ April 6

Name: Solutions

Recitation (circle yours): Klobusicky (TR 2:30)   Mogin (TR 3:30)

Note: No calculators or electronic devices of any sort are allowed. Show all work necessary to obtain your answers. No credit will be given for answers without justification.

3 (20 points) You wish to build a box with a square base and no top. You have 12 cm^2 of material to work with. What is the largest volume box you can make?



$$V = x^2 h$$

$$SA = x^2 + 4xh = 12$$

$$4xh = 12 - x^2$$

$$h = \frac{12}{4x} - \frac{x^2}{4x} = \frac{3}{x} - \frac{x}{4}$$

$$V = x^2\left(\frac{3}{x} - \frac{x}{4}\right) = 3x - \frac{x^3}{4}$$

$$\frac{dV}{dx} = 3 - \frac{3x^2}{4} = 0 \quad \text{if} \quad 3 = \frac{3x^2}{4}$$

$$4 = x^2$$

$$2 = x$$

$V'' = -\frac{3x}{2} < 0$ for $x > 0$ so $2 = x$ is abs. max.

Largest volume has $x = 2$, $h = \frac{3}{2} - \frac{2}{4} = 1$

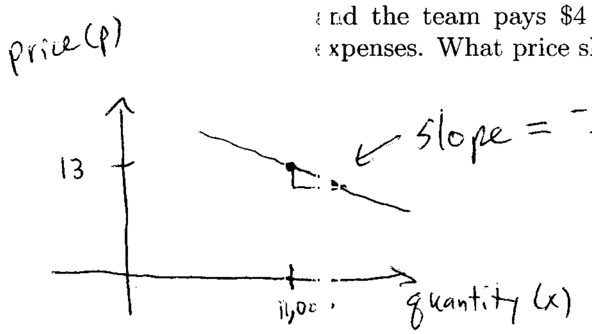
$$\boxed{V = 2^2 \cdot 1 = 4 cm^3}$$

4 (20 points) A hockey team plays in an arena with seating capacity of 15,000. Charging a ticket price of \$13, they have an average attendance of 11,000 people per game. Their market research indicates that for every dollar they reduce the price of a ticket, attendance will increase by 1000 people per game.

(a) Find the demand function, assuming it is linear.

(b) What price should they charge to maximize revenue?

(c) Assume there are fixed costs of \$40,000 per game to open the arena, and the team pays \$4 per ticket-buyer for security and janitorial expenses. What price should they charge to maximize their profit?



a) demand equation:

$$p - 13 = \frac{-1}{1000}(x - 11{,}000)$$

$$= \frac{-x}{1000} + 11$$

$$\boxed{p = -\frac{x}{1000} + 24}$$

b) $R(x) = xp(x) = x\left(\frac{-x}{1000} + 24\right) = \frac{-x^2}{1000} + 24x$

$R'(x) = \frac{-x}{500} + 24 = 0$ if $\frac{x}{500} = 24$

$x = 12{,}000$

$R'' = \frac{-1}{500} \Rightarrow x = 12{,}000$ is abs. max.

They should charge $p(12{,}000) = -\frac{12{,}000}{1000} + 24 = -12 + 24 = \boxed{12 \text{ dollars}}$

c) Cost function $C(x) = 40{,}000 + 4x$

Profit $P(x) = R(x) - C(x) = \frac{-x^2}{1000} + 24x - (40{,}000 + 4x) = \frac{-x^2}{1000} + 20x - 40{,}000$

$P' = \frac{-x}{500} + 20 = 0$ if $\frac{x}{500} = 20$, $\boxed{x = 10{,}000}$

So to maximize profit, they should charge $p(10{,}000) = \frac{-10{,}000}{1000} + 24 = \boxed{\$14}$

5 (20 p[illegible]ints) A stone is dropped into a calm body of water, causing a circular rippl[illegible] The area of the ripple grows at a rate of 2 cm^2 per second. Find the r[illegible]te of change of the radius when the radius is 5 cm.

$$A = \pi r^2 \qquad \frac{dA}{dt} = 2 \qquad \left.\frac{dr}{dt}\right|_{r=5} = ?$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$2 = 2 \cdot \pi \cdot 5 \left.\frac{dr}{dt}\right|_{r=5}$$

$$\boxed{\frac{1}{5\pi} = \left.\frac{dr}{dt}\right|_{r=5}}$$