HW9

4.1

2. $27^x = (3)^{3x}$ $(\sqrt[3]{2})^x = (2)^{x/3}$ $\left(\frac{1}{8}\right)^x =$ ~~$(2)^{x/3}$~~
$= (2)^{-3x}$

4. $9^{-x/2} = 3^{-x}$ $8^{4x/3} = 2^{4x}$ $27^{-2x/3} = 3^{-2x}$

10. $\frac{2^x}{6^x} = \frac{2^x}{2^x 3^x} = \frac{1}{3^x} = \left(\frac{1}{3}\right)^x = 3^{-x}$

$\frac{3^{-5x}}{3^{-2x}} = \frac{3^{-2x}3^{-3x}}{3^{-2x}} = 3^{-3x}$

$\frac{16^x}{8^{-x}} = 16^x 8^x = 2^{4x} 2^{3x} = 2^{7x}$

14. $(3^{-x} \cdot 3^{x/5})^5 = 3^{-5x} 3^x = 3^{-4x}$

$(16^{1/4} \cdot 16^{-3/4})^{3x} = 16^{3x/4} \cdot 16^{-9x/4} = 16^{-6x/4} = 16^{-3x/2}$
$= 2^{-12x/2}$
$= 2^{-6x}$

18. $10^{-x} = 10^2$
$-x = 2$
$x = -2$

26. $(3^{2x} \cdot 3^2)^4 = 3$
$(3^{8x} \cdot 3^8) = 3$
$3^{8x+8} = 3^1$

$8x + 8 = 1$
$8x = -7$
$x = \frac{-7}{8}$

38. $5^{2+h} = 25(\quad)$

$\cancel{5^{2+h} = 5^{2}(1+h/2)}$ oops!

$5^2 5^h = 25(5^h)$

40. $5^{x+h} + 5^x = 5^x 5^h + 5^x = 5^x(5^h + 1)$

4.2 7. $h = 0.01$: $\dfrac{e^h - 1}{h} = \dfrac{e^{0.01} - 1}{0.01} \approx 1.005$

$h = 0.001$ $\dfrac{e^{0.001} - 1}{0.001} \approx 1.001$

$h = 0.0001$ $\dfrac{e^{0.0001} - 1}{0.0001} \approx 1.000$

(The point is that it gets more accurate as $h \to 0$)

26. $y = 7e^x - 2x^2$ $\dfrac{dy}{dx} = 7e^x - 4x$

28. $y = \dfrac{1}{1+e^x} = (1+e^x)^{-1}$

$\dfrac{dy}{dx} = -1\left((1+e^x)^{-2}(e^x)\right) = \dfrac{-e^x}{(1+e^x)^2}$

30. $y = \dfrac{x}{e^x}$ $\dfrac{dy}{dx} = \dfrac{e^x(1) - x(e^x)}{(e^x)^2} = \dfrac{e^x - xe^x}{e^{2x}}$

$= \dfrac{1-x}{e^x}$

32. $y = \frac{e^x}{x^2}$ $\frac{dy}{dx} = \frac{x^2(e^x) - e^x(2x)}{(x^2)^2}$

$$= \frac{x^2e^x - 2xe^x}{x^4} = \frac{xe^x - 2e^x}{x^3}$$

34. $y = (1+x^2)e^x \Rightarrow \frac{dy}{dx} = e^x(2x) + (\cancel{*}1+x^2)(e^x)$

$$= 2xe^x + e^x + x^2e^x = e^x(x^2+2x+1)$$

36. $y = (xe^x - 1)^{-3}$

$\frac{dy}{dx} = -3(xe^x-1)^{-4}(e^x + xe^x)$ $\cancel{= 2e^x + 2e^x}$

$$= \frac{-3e^x(1+x)}{(xe^x-1)^4}$$

40. $y = xe^x$ $\left.\frac{dy}{dx}\right|_{(1,e)} = e^1 + (1)e^1 = 2e$

$\frac{dy}{dx} = e^x + xe^x$

4.3 2. $f(x) = 3e^7$ $f'(x) = 0$

4. $f(x) = \frac{e^{3x}}{3}$ $f'(x) = \frac{1}{3}(3)e^{3x} = e^{3x}$

6. $f(t) = \cancel{e}\,\frac{1}{e^{-t}} = e^t$ $f'(t) = e^t$

8. $f(x) = 2e^{\sqrt{x}} = 2e^{(x)^{1/2}}$

$$f'(x) = 2e^{x^{1/2}}\left(\frac{1}{2}x^{-1/2}\right) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$$

10. $g(x) = (e^{-2x} - 2x)^3$

$$g'(x) = 3(e^{-2x} - 2x)^2(-2e^{-2x} - 2)$$

$$= -6(e^{-2x} - 2x)^2(e^{-2x} + 1)$$

12. $y = \dfrac{e^{-2x^2+2x+1}}{10}$

$$\cancel{y' = \frac{1}{10}(1-4x)e^{-2x^2}}$$

$$y' = \frac{e^{-2x^2+2x+1}}{10}\,\frac{d}{dx}\left[-2x^2 + 2x + 1\right]$$

$$= \frac{e^{-2x^2+2x+1}}{10}(-4x + 2)$$

$$= \frac{(1-2x)e^{-2x^2+2x+1}}{5}$$

14. $g(t) = t^2 e^{1/t}$

$g'(t) = 2te^{1/t} + \frac{d}{dt}\left(e^{t^{-1}}\right) t^2$

$= 2te^{1/t} + \frac{-1}{t^2}\left(e^{t^{-1}}\right) t^2$

$= e^{1/t}(2t-1)$

4.4 2. $\ln\left(\frac{1}{e^2}\right) = \ln(e^{-2}) = -2$

3. $e^x = 5$

~~$\log e^x = \log 5$~~ $\ln e^x = \ln 5$

$x \ln e = \ln 5$

$x = \ln 5$

4. $e^{-x} = 3.2$

$\ln e^{-x} = \ln 3.2$

$-x \ln e = \ln 3.2$

$-x = \ln 3.2$

$x = \ln(3.2)^{-1}$

$x = \ln\left(\frac{1}{3.2}\right)$

6. $\ln x = 4.5$

$e^{\ln x} = e^{4.5}$

$x = e^{4.5}$

7. $\ln e^{-3} = -3 \ln e = -3$

8. $e^{\ln 4.1} = 4.1$

16. $\ln(e^{-2}e^{4}) = \ln(e^{2}) = 2\ln e = 2$

18. $e^{\ln 3 - 2\ln x} = e^{\ln 3 - \ln x^2} = \dfrac{e^{\ln 3}}{e^{\ln x^2}} = \dfrac{3}{x^2}$

20. $e^{1-3x} = 4$

$\ln e^{1-3x} = \ln 4$

$1-3x = \ln 4$

$3x = 1 - \ln 4 = 1 + \ln \frac{1}{4}$

$x = \dfrac{1 + \ln \frac{1}{4}}{3}$

22. $\ln 3x = 2$

$e^{\ln 3x} = e^{2}$

$3x = e^{2}$

$x = \dfrac{e^{2}}{3}$