HW 8 Solutions

Sect 3.1

32) Points of Inflection occur at $g'' = 0$

$$g' = \frac{-2x}{(x^2+1)^2} \qquad g'' = \frac{(x^2+1)^2 \cdot -2 - (-2x)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4} = 0$$

$$\Rightarrow (x^2+1)\cdot -2 - (-2x)\cdot 2\cdot 2x = 0$$

$$\Rightarrow -2x^2 - 2 + 8x^2 = 0 \Rightarrow 6x^2 = 2 \Rightarrow x^2 = \frac{1}{3} \text{ so } x = \pm\sqrt{\frac{1}{3}}.$$

66) $\frac{d}{dx}[f(x)g(x)]\Big|_{x=1} = f(1)g'(1) + g(1)f'(1)$ by the product rule

$= 2\cdot 5 + 4\cdot 3 = 22.$

61) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\Big|_{x=1} = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2} = \frac{4(3) - 2(5)}{16} = \frac{1}{8}.$

Sect 3.2

48) $g = \frac{x}{\sqrt{2-x^2}}$ $g' = \frac{\sqrt{2-x^2}(1) - x(-2x)\frac{1}{2\sqrt{2-x^2}}}{2-x^2}$

At $x=1$ $g' = \frac{\sqrt{2}+1}{1} = \sqrt{2}+1$

For $g = mx+b$ Thus $g = (\sqrt{2}+1)x - \sqrt{2}$

$1 = (\sqrt{2}+1)(1) + b$

$b = -\sqrt{2}$

59) $\frac{d}{dx} f(g(x))\Big|_{x=1} = g'(1)(f'(g(1)))$ By the chain rule

$= 6(f'(5)) = 6 \cdot 4 = 24$

60) $\frac{d}{dx} g(f(x))\Big|_{x=1} = f'(1)(g'(f(1))) = 3(g'(2)) = 3 \cdot 7 = 21.$

33. 2) $x^3 + y^3 - 6 = 0$

$\frac{d}{dx}(x^3 + y^3 = 1)$

$3x^2 + 3y^2 \frac{dy}{dx} = 0$

$3y^2 \frac{dy}{dx} = -3x^2$

$\frac{dy}{dx} = -\frac{x^2}{y^2}$

10) $xy^3 = 2$ By the product rule,

$x \cdot 3y^2 \frac{dy}{dx} + y^3 = 0$

$\frac{dy}{dx} = \frac{-y^3}{x \cdot 3y^2} = -\frac{y}{3x}$

22) $\sqrt{x} + \sqrt{y} = 7$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

At $x = 9, y = 16$; $\frac{dy}{dx} = -\frac{4}{3}$

27) $x^4 + 2x^2y^2 + y^4 = 4x^2 - 4y^2$

$4x^3 + 2(x^2 \cdot 2y \frac{dy}{dx} + y^2 \cdot 2x) + 4y^3 \frac{dy}{dx} = 8x - 8y \frac{dy}{dx}$

$4x^3 + 4xy^2 - 8x = (-4y^3 - 8y - 4x^2y) \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{4x^3 + 4xy^2 - 8x}{-(4y^3 + 8y + 4x^2y)} = \frac{4\left(\frac{6\sqrt{6}}{8}\right) + 4\left(\frac{\sqrt{6}}{2}\right)\left(\frac{1}{2}\right) - 8\left(\frac{\sqrt{6}}{2}\right)}{-\left(4\left(\frac{2\sqrt{2}}{8}\right) + 8\left(\frac{\sqrt{2}}{2}\right) + 4\left(\frac{3}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\right)}$

$= \frac{3\sqrt{6} + \sqrt{6} - 4\sqrt{6}}{-(\sqrt{2} + 4\sqrt{2} + 3\sqrt{2})} = 0$

$y = mx + b, \quad \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2}(0) + b, \quad y = \frac{\sqrt{2}}{2}.$

32) $y^4 - x^2 = 1$

$\frac{d}{dx}[y^4 - x^2 = 1]$

$\Rightarrow 4y^3 \frac{dy}{dx} - 2x \frac{dx}{dx} = 0$

$\frac{dy}{dx} = 2x \frac{dx}{dx} / 4y^3 = \frac{x \frac{dx}{dx}}{2y^3}$

34) $y^2 = 8 + xy$ $\quad \frac{d}{dt}[y^2 = 8 + xy]$

$\Rightarrow 2y\frac{dy}{dt} = 0 + x\frac{dy}{dt} + y\frac{dx}{dt}$

$(2y - x)\frac{dy}{dt} = y\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{y\frac{dx}{dt}}{2y - x}$

44) $V = \frac{\pi x^3}{6}$ $\quad \frac{dV}{dt} = \frac{dV}{dx}\cdot\frac{dx}{dt}$ $\quad \frac{dx}{dt} = .4\ mm/s$

$\frac{dV}{dx} = \frac{\pi x^2}{2}$ $\quad$ at $x = 10$ $\frac{dV}{dx} = 50\pi$

Thus $\frac{dV}{dt} = (50\pi\ mm^2)(.4\ mm/s) = 20\ mm^3/s.$

45) a) $x^2 + y^2 = 100$

b) $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\quad 8^2 + y^2 = 100 \Rightarrow y^2 = 36$ $y = 6.$

$\frac{dy}{dt} = -\frac{x\frac{dx}{dt}}{y} = \frac{-8(3)}{6} = -4$

46) a) $x^2 + 5000^2 = y^2$

b) $x^2 = (13000)^2 - 5000^2 = 1000^2(169 - 25) = 1000^2(144)$

$x = 1000(12) = 12000.$

c) $2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$\frac{2(12000)(\quad)}{2(13000)} = \frac{dy}{dt} = \frac{12(550)}{13} - 12(30) = 360.$