HW 5 SOLUTIONS

Sect. 1.4

61) $\lim_{x\to\infty} \frac{1}{x^2} = \lim_{x\to\infty} \frac{1}{x} \cdot \lim_{x\to\infty} \frac{1}{x} = 0 \cdot 0 = 0$

63) $\lim_{x\to\infty} \frac{1}{x-8} = \frac{1}{\lim_{x\to\infty} x-8} = \frac{1}{\infty} = 0.$

64) $\lim_{x\to\infty} \frac{5x+3}{3x-2} = \lim_{x\to\infty} \frac{5x}{3x} = \frac{5}{3}.$

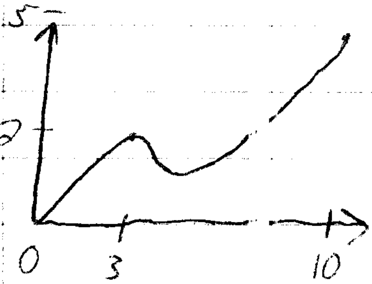
65) $\lim_{x\to\infty} \frac{10x+100}{x^2-30} = \lim_{x\to\infty} \frac{10x}{x^2} = \lim_{x\to\infty} \frac{10}{x} = 0.$

66) $\lim_{x\to\infty} \frac{x^2+x}{x^2-1} = \lim_{x\to\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} 1 = 1.$

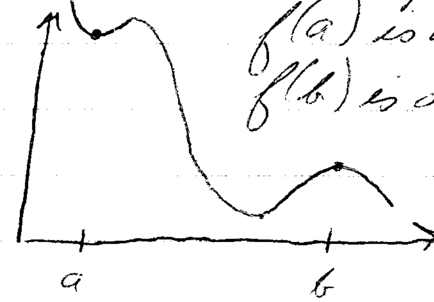
Sect 2.1

31) At $t=60$, the graph has the steepest downward slope. Thus, at $t=60$, farms decrease more rapidly, or in $60+1900 = \underline{1960}$

32) At $t=2$, the slope is smallest thus in $1983+2=1985$, the rate of increase is the smallest. At $t=16$, thus in $1983+16=1999$, the rate is increase is at its greatest, as the slope is steepest.

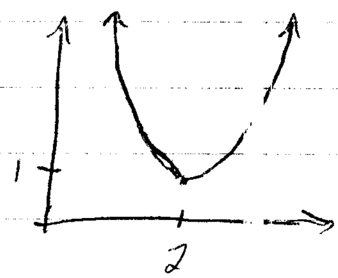
35) 

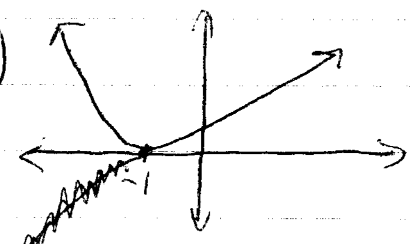
40) No, consider this graph:



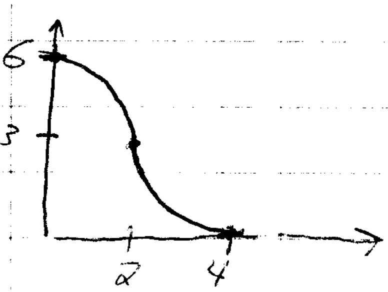
$f(a)$ is a relative min.
$f(b)$ is a relative max.
$f(a) > f(b)$
Contradiction.

<u>Sect 2.2.</u>

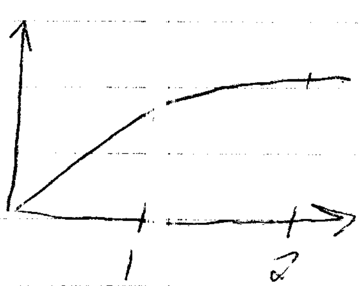
7) 

8) 

SORRY!

11) 

~~22) v(1) > v(2), the velocity at one is greater than the velocity at 2, then the car is going faster at t=1 than t=2.~~

~~22)~~ 21) 

$f'(1) > f'(2)$, thus the car is moving faster at 1.

22) $v(1) < v(2)$, thus the car is faster at $t=2$.

26) At $x=4$, $f'(x)$ is negative; a negative slope implies a decreasing function

28) At $x=5$ $f'(x)$ is 0. Thus there is an extrema. $f''(5)$ is positive since the slope at 5 is positive. $f'(x)=0$ and $f''(x)>5$ implies a minimum is attained.

30) At $x=2$ $f''(x)$ is negative, which means $f(2)$ is concave down.

32) At $x=4$ $f''(x)=0$, which implies an inflection point.

33) $f(6)=3$ $f'(6)=2$ $y=mx+b$

$m=2$ $3=(2)(6)+b$

$b=-9$ Thus $y=2x-9$

34) Find the tangent line $f'(6)=2$, $f(6)=8$ $y=mx+b$

$8=(2)(6)+b$

$b=-4$

Then $y=2x-4$, which approximates f near 6. Thus $f(6.5)\approx 2(6.5)-4=9$.

Sect 2.4

7) $f(x) = \frac{1}{3}x - 2x^2 + 5x$
$f'(x) = x^2 - 4x + 5$
When $f'(x) = 0$ $x = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2}$

Thus there are no real roots, or no relative extrema.

8) $f(x) = -x^3 + 2x^2 - 6x + 3$
$f'(x) = -3x^2 + 4x - 6$
If $x < 0$, both $-3x^2$ and $4x$ are negative, so if $x<0$, $f'(x) < 0$
Also if $f'(x) = 0$ $x = \frac{-4 \pm \sqrt{16-4(6)(3)}}{-6}$, which is not real.

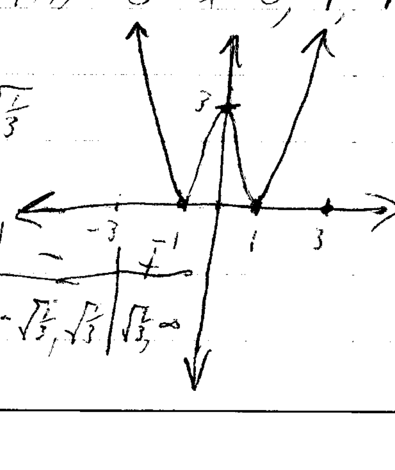
Thus $f'(x)$ is never 0, which means that it can never become positive, since it would have to be 0 at some point first.

20) $f(x) = 3x^4 - 6x^2 + 3$
$f'(x) = 12x^3 - 12x$ $0 = 12x^3 - 12x \Rightarrow x^3 - \cancel{12}x = 0$
$x(x^2 - 1) = 0$
$x(x-1)(x+1) = 0$ $x = 0, -1, +1$

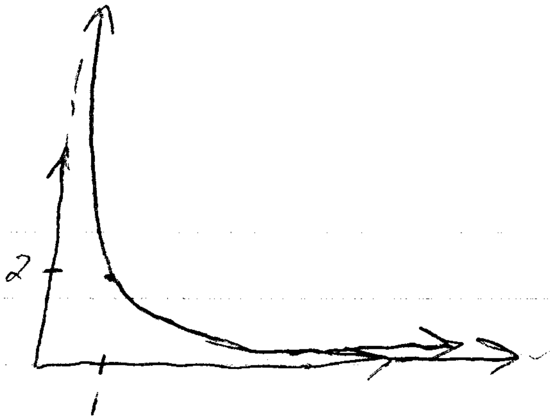
$f''(x) = 36x^2 - 12$
$0 = 36x^2 - 12 \Rightarrow 0 = 3x^2 - 1 \Rightarrow x = \sqrt{\tfrac{1}{3}}, -\sqrt{\tfrac{1}{3}}$

$f'(x)$	$-$	$+$	$-$	$+$
x	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty$

$f''(x)$	$+$	$-$	$+$
x	$-\infty, -\sqrt{\tfrac13}$	$-\sqrt{\tfrac13}, \sqrt{\tfrac13}$	$\sqrt{\tfrac13}, \infty$



24) $g = 2/x$



$\lim_{x\to\infty} \frac{2}{x} = 0$ → Hor. Asymptote at $g=0$

At $x=1$, $g = \frac{2}{1} = 2$

$\lim_{x\to 0} \frac{2}{x} = \infty$ Vertical Asymptote at $x=0$

26) $g = \frac{12}{x} + 3x + 1$

$g' = -\frac{12}{x^2} + 3$ $g' = 0 = -\frac{12}{x^2} + 3 \Rightarrow +\frac{12}{x^2} = 3 \Rightarrow x^2 = 4 \Rightarrow x = -2, 2.$

As $x \to \infty$ $f(x)$ mimics $3x+1$

As $x \to 0^+$ $f(x) \to \infty$

As $x \to 0^-$ $f(x) \to -\infty$

At $x=2$ $f(x) = 13$

$x=-2$ $f(x) =$