Solutions HW 2.

Sec 1.1

6) $2x + 7y = -1$

$7y = -1 - 2x$

$y = -\frac{1}{7} - \frac{2}{7}x$

$m = -\frac{2}{7} \quad b = -\frac{1}{7}$

22) $x + y = 0 \Rightarrow y = -x \quad m = -1$

$y = mx + b \Rightarrow y = -1x + b$

Using $(1,1)$ $\quad 1 = -1(1) + b \Rightarrow b = 2$

$y = -x + 2$

34) $m = \frac{\Delta y}{\Delta x} = \frac{2}{\cancel{2}} \quad \Delta x = \frac{1}{4}$

$\Rightarrow \Delta y = 2 \cdot \frac{1}{4} = \frac{1}{2}$

47) Let X = time in ~~years~~ months from January, 2004

Let Y = Gasoline price

m = Price increase/month = .06

b = Price at 0 months = 1.69

$Y = mx + b = .06x + 1.69$

At $x = 3$ (April) $\quad Y = .06(3) + 1.69 = 1.87 \times 15 = 28.05$ for 15 gallons

$x = 8$ (Sept) $\quad Y = .06(8) + 1.69 = 2.17 \times 15 = 32.55$ for 15 gallons.

58) Let Y = Degrees Celsius

Let X = Degrees Fahrenheit

$m = \frac{\Delta y}{\Delta x} = \frac{100 - 0}{212 - 32} = \frac{100}{180} = \frac{5}{9}$

$y = \frac{5}{9}x + b \quad 0 = \frac{5}{9}(32) + b \Rightarrow b = -\frac{160}{9}$

Then $y = \frac{5}{9}x - \frac{160}{9}$ $\quad$ At $x = 98.6$ $\quad y = \frac{5}{9}(98.6) - \frac{160}{9} = 37°C$

Sect 1.2

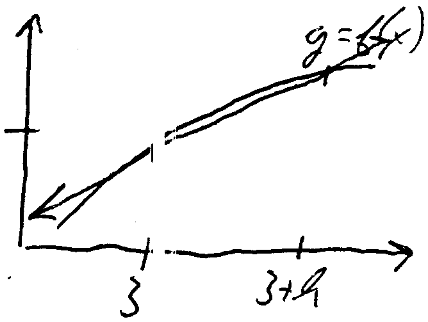
5) $m = \cancel{\frac{\Delta x}{\Delta y} = \frac{3 \text{ units}}{}} = \frac{\Delta y}{\Delta x} = \frac{-4 \text{ units}}{+3 \text{ units}} = \frac{-4}{3}$

6) $m = \frac{\Delta y}{\Delta x} = \frac{0 \text{ unit}}{1 \text{ unit}} = 0$

(Note: for Δx you can substitute any number, as Δy remains 0.)

7) $m = \dfrac{\Delta y}{\Delta x} = \dfrac{1 \text{ unit}}{1 \text{ unit}} = 1$ 8) $m = \dfrac{\Delta y}{\Delta x} = \dfrac{1 \text{ unit}}{1 \text{ unit}} = 1.$

38) e.



$m = \dfrac{\Delta y}{\Delta x} = \dfrac{f(3+h) - f(3)}{3+h-3} = \dfrac{f(3+h) - f(3)}{h}$

<u>Sect 1.4</u> 30) $f'(2) = \lim\limits_{h \to 0} \dfrac{(2+h)^3 - 2^3}{h}$

$= \lim\limits_{h \to 0} \dfrac{8 + 12h + 6h^2 + h^3 - 8}{h} = \lim\limits_{h \to 0} \dfrac{12h + 6h^2 + h^3}{h} = \lim\limits_{h \to 0} 12 + 6\overset{0}{h} + \overset{0}{h}{}^2 = 12$

49) Let $f(x) = x^2$ and $a = 1$

$f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \cancel{(1+h)^2 - 1} \lim\limits_{h \to 0} \dfrac{(1+h)^2 - 1}{h}$

50) Let $f(x) = x^3$ and $a = 2$

$f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \cancel{(2+h)^3 - 8} \lim\limits_{h \to 0} \dfrac{(2+h)^3 - 8}{h}.$

53) Let $f(x) = \sqrt{x}$ and $a = 9$

$f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h \to 0} \dfrac{\sqrt{9+h} - 3}{h}$

54) Let $f(x) = x^{-\frac{1}{2}}$ and $a = 1$

$f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h \to 0} \dfrac{(1+h)^{-\frac{1}{2}} - (1)^{-\frac{1}{2}}}{h} = \lim\limits_{h \to 0} \dfrac{(1+h)^{-\frac{1}{2}} - 1}{h}$

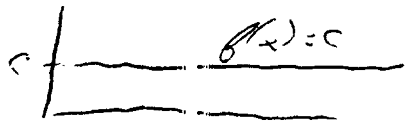
Additional Exercises

1) To approximate f at 2, we construct the tangent line at $x=2$.
For m, $\frac{\Delta y}{\Delta x} = 7$ at $x=2$, since $f'(2)=7$.

Then $y = mx+b = 7x+b$. Since $(2,4)$ is a point,
$4 = 7(2)+b \Rightarrow b=-10$

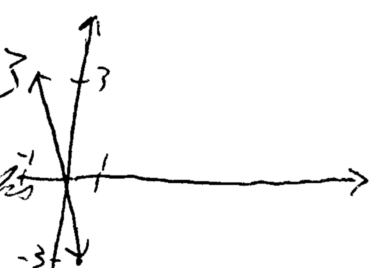
Then $y = 7x-10$.
So near 2, $f(x) \approx 7x-10$
Thus $f(2.001) \approx 7(2.001)-10 = 4.007$.
$f(1.999) \approx 7(1.999)-10 = 13.993$.

2) Let f be constant! For $f(x)=c$, where c is a constant, the entire graph is horizontal, thus slope 0.



3) Let $f(x) = -3x$. Then $f'(x) = -3$

Note: $f(x) = -3x+c$, c constant, also works.



4) False. Counterexample: Let $f(x) = 5x+1$, $g(x) = 2x+100$.
$f'(2) = 5 > 2 = g'(2)$, yet $f(2) = 11 < g(2) = 104$.

5) True. If $f'(x) > 0$ for all x, then f is increasing since its slope is positive. Then if $x>y$, $f(x) > f(y)$, which means that $f(2) > f(1)$.