

HOMEWORK 11

4.4) 4) $f(x) = 5x - 2e^x$

$f'(x) = 5 - 2e^x$ $5 - 2e^x = 0 \Rightarrow \frac{5}{2} = e^x \Rightarrow x = \ln \frac{5}{2}$

$f''(x) = -2e^x < 0$, so it is a max. $(x, y) = (\ln \frac{5}{2}, 5 \ln \frac{5}{2} - 5)$

4.7) $2^x = e^{kx}$

$x \ln 2 = kx$

$\ln 2 = k$

4.5) 6) $y = \ln [e^{x^2} + 2]$

$y' = (e^{x^2} + 2)' \cdot \frac{1}{e^{x^2} + 2}$

$= \frac{2x(e^{x^2})}{e^{x^2} + 2} = \frac{2xe^{x^2}}{e^{x^2} + 2}$

4.6) 76) $\ln x^2 - \frac{1}{2} \ln 2x + 1 = 0$

$2 \ln x - (\ln 2 + \ln x) = -1$

$\ln x - \ln 2 = -1$

$\ln x = \ln 2 - 1$

$x = e^{-\ln 2 - 1}$

78) $3 \ln x - \ln 3x = 0$

$3 \ln x - \ln 3 - \ln x = 0$

$2 \ln x = \ln 3$

$\ln x^2 = \ln 3$

$x^2 = 3$

$x = \sqrt{3}$

32) $\ln \left[\frac{(x-3)}{(x+2)} \right] - \ln (x+2)^2 - \ln 7 = 0$

$\ln(x-3) + \ln(x+2) - 2 \ln(x+2) - \ln 7 = 0$

$\ln(x-3) - \ln(x+2) - \ln 7 = 0$

$\ln \frac{(x-3)}{(x+2)} = 0 \Rightarrow \frac{(x-3)}{(x+2)} = 1$

$(x-3) = 7x + 14$

$-17 = 6x$

$x = -\frac{17}{6}$

$$5.1) 4) P(t) = 300e^{.01t}$$

$$a) t=0, \text{ so } P(0) = 300e^0 = 300$$

$$b) P'(t) = .01P(t)$$

$$c) 600 = 300e^{.01t}$$

$$2 = e^{.01t}$$

$$\ln 2 = .01t$$

$$t = 69.$$

$$d) 1200 = 300e^{.01t}$$

$$4 = e^{.01t}$$

$$2 \ln 2 = .01t$$

$$t = 138.6$$

$$6) 3 = 1e^{kt}$$

$$\ln 3 = k$$

$$k = \ln 3 / 10.$$

$$8) 2 = e^{.04t}$$

$$\ln 2 = .04t$$

$$\frac{\ln 2}{.04} = t$$

$$.04$$

$$t = 27.4653$$

$$10) P(t) = 5.51e^{kt} \text{ when } 0 = 1995$$

$$5.88 = 5.51e^{5k}$$

$$1.06715 = e^{5k}$$

$$\ln 1.06715 = 5k$$

$$k = .0123$$

$$P(7) = 5.51e^{(.0123)7}$$

$$1.27042 = e^{t(.0123)}$$

$$\ln(1.27042) = .0123t$$

$$t = 19.459$$

The year 2012.46.

$$16) P(t) = 10e^{-.4t}$$

$$a) P'(t) = -.4P(t)$$

$$c) P(5) = 10e^{-.4(5)}$$

$$= 8.187$$

$$20) 3 = 10e^{5k}$$

$$\ln 3 = 5k$$

$$k = -.2408$$

$$\frac{1}{2} = e^{t(-.2408)}$$

$$\ln \frac{1}{2} = -.2408t$$

$$t = 2.8775$$

$$25) \frac{1}{5} = e^{-.0012t}$$

$$\ln \frac{1}{5} = -.0012t$$

$$t = 1341.2$$

52) 4) a) $A'(t) = .046A(t)$

b) $A(t) = 10000e^{.046t}$

c) $A(3) = 14779.8$

d) ~~8~~ $A'(2) = .046(30,000) = 1380/yr$

10) $P(t) = 20,000e^{rt}$
 $29,140,000 = 20,000e^{r(149)}$
 $1389.83 = e^{49r}$
 $r = .1448$

20) $P(10) = 20000e^{.08(10)}$
 $= 4451.08$

54) a) $g(x) = 10 - 10e^{-.1x} \quad x \geq 0$

$g'(x) = +10e^{-.1x} \geq 10 > 0 \quad x \geq 0$

$g''(x) = -10e^{-.1x} < 0 \quad x \geq 0$

b) As x gets large $e^{-.1x} \rightarrow 0$, so $10 - 10e^{-.1x} \rightarrow 10$



4) $g = 5(1 - e^{-2x}) \quad g' = 5(2e^{-2x}) = 10e^{-2x}$

~~10(3) = 10 - 2g = 10 - 2(5 - e^{-2x}) = 10e^{-2x} = g'~~

~~Additional Equations: $500(1.04)^{360} - 1 = 347024.70$~~

~~1) $P(30) = 500(1.04)^{30} = 1066.75$~~

~~2) Using $M = PR / (1+R)^n$
 $500 = P(0.04) / (1.04)^{30}$
 $P = 500(1+1.04)^{30} = 8646$~~

~~3) $P(30) = 104730.62 (1 + \frac{.04}{12})^{12 \cdot 30} = \frac{1686.75}{500} \cdot 105730.62 = 347024.91$~~

~~4) $M = \frac{11(12030)}{1-1.11} = 3867.92$~~

ing for N gives $N = \frac{-\ln(1 - \frac{M}{PR})}{\ln(AR)} = \frac{-\ln(1 - \frac{150}{10000(.06)})}{\ln(1.06)} = 4937$, so 5 months.

Additional Exercises

$$\begin{aligned} 1) \text{ Total amount} &= 500 \left(1 + \left(1 + \frac{0.04}{12} \right)^{12} \right)^{360} - \left(1 + \frac{0.04}{12} \right)^{360} \\ &= 500 \frac{\left(1 + \frac{0.04}{12} \right)^{360} - 1}{\frac{0.04}{12}} = 347024.70 \end{aligned}$$

$$2) M = \frac{PR}{1 + \frac{k}{12}} \cdot \frac{500 \left(1 + \left(1 + \frac{0.04}{12} \right)^{360} \right)}{\frac{0.04}{12}} = 104,730.60$$

$$3) P(30) = 104,730.60 \left(1 + \frac{0.04}{12} \right)^{12(30)} = 347024.91$$

$$4) M = \frac{.11(120000)}{1 - (1.1)^{-4}} = 3867.90$$

$$5) \text{ Solving for } N \text{ gives } N = \frac{-\ln \left(1 - \frac{M}{PR} \right)}{\ln(1+R)} = \frac{-\ln \left(1 - \frac{150}{12000(1.08)} \right)}{\ln(1.08)} = 4.937$$