HW 10 solutions

Section 4.1

16) $8^{-x/3} = (8^{-1/3})^x = \left(\frac{1}{8^{1/3}}\right)^x = \left(\frac{1}{2}\right)^x$ $\boxed{b = \frac{1}{2}}$

28) $3^{5x} 3^{x} - 3 = 0$

$3^{5x+x} - 3 = 0$

$3^{6x} = 3$

$6x = 1$

$\boxed{x = \frac{1}{6}}$

34) $2^{2x+2} - 17 \cdot 2^x + 4 = 0$

$2^{2x} \cdot 2^2 - 17 \cdot 2^x + 4 = 0$

$4(2^x)^2 - 17 \cdot 2^x + 4 = 0$

Let $y = 2^x$:

$4y^2 - 17y + 4 = 0$

Quadratic formula: $y = \frac{17 \pm \sqrt{289 - 64}}{8} = \frac{17 \pm 15}{8} = \frac{2}{8}$ or $\frac{32}{8}$

$y = \frac{1}{4}$ or $y = 4$

So $2^x = \frac{1}{4} \Rightarrow x = -2$

or $2^x = 4 \Rightarrow x = 2$

$\boxed{x = 2 \text{ and } x = -2}$ are the solutions to the equation

Section 4.2

#37) $y = (1+x^2)e^x$

Tan. line horizontal $\Longleftrightarrow$ $y' = 0$

$$y' = (1+x^2)e^x + e^x(2x) = 0$$

$$e^x(1+x^2+2x) = 0$$

$e^x > 0$ for all x, so this is only true if

$$x^2+2x+1=0$$

$$\cancel{e}^x(x+1)(x+1) = 0 \quad \Rightarrow \quad \boxed{x=-1}$$

$$y(-1) = (1+(-1)^2)e^{-1} = \frac{2}{e}$$

The tan. line is horizontal at the point $\left(-1, \frac{2}{e}\right)$

#42) $y = \dfrac{e^x}{x+e^x}$

$$y' = \frac{(x+e^x)e^x - e^x(1+e^x)}{(x+e^x)^2}$$

$$y'(0) = \frac{(0+1)\cdot 1 - 1(1+1)}{(0+1)^2} = \frac{1-2}{1} = -1$$

slope of tan. line

equation of tan. line:

$$y-1 = -1(x-0)$$

$$y-1 = -x$$

$$\boxed{y = -x+1}$$

Section 4.3

#20) $f(x) = \sqrt{e^{x/2} + 1} = (e^{x/2}+1)^{1/2}$

$$f'(x) = \frac{1}{2}(e^{x/2}+1)^{-1/2} \cdot \frac{1}{2}e^{x/2}$$

$$\boxed{= \frac{e^{x/2}}{4\sqrt{e^{x/2}+1}}}$$

#24) $f(x) = e^{ex}$

$$f'(x) = e\,e^{ex} \qquad (= e^{ex+1})$$

Section 4.4

#28) $\ln(x^2-5) = 0$

$$x^2 - 5 = e^0 = 1$$
$$x^2 = 6$$
$$\boxed{x = \pm\sqrt{6}}$$

#30) $2\ln x = 7$

$$\ln x = 7/2$$
$$x = e^{7/2}$$

#34) $750e^{-.4x} = 375$

$$e^{-.4x} = \frac{1}{2}$$
$$-.4x = \ln\left(\tfrac{1}{2}\right) = -\ln 2$$
$$.4x = \ln 2$$
$$\boxed{x = \frac{\ln 2}{.4} = \frac{5\ln 2}{2}}$$

#36) $e^{5x}e^{\ln 5} = 2$

$$e^{5x} \cdot 5 = 2$$
$$e^{5x} = \frac{2}{5}$$
$$5x = \ln\left(\tfrac{2}{5}\right)$$
$$\boxed{x = \frac{\ln\left(\frac{2}{5}\right)}{5}}$$

#38) $(e^x)^2 e^{2-3x} = 4$

$$e^{2x}e^{2-3x} = 4$$
$$e^{2x+2-3x} = 4$$
$$e^{2-x} = 4$$
$$2 - x = \ln 4$$
$$-x = \ln 4 - 2$$
$$\boxed{x = 2 - \ln 4}$$

#40) $f(x) = -1 + (x-1)^2 e^x$

$f'(x) = (x-1)^2 e^x + e^x(2(x-1))$

$= e^x((x-1)^2 + 2x - 2)$

$= e^x(x^2 - 2x + 1 + 2x - 2)$

$= e^x(x^2 \; -1)$

$= e^x(x+1)(x-1) = 0$ if $x = 1$ or $x = -1$

$f(1) = -1 + (1-1)^2 e^1 = -1$

$f(-1) = -1 + (1-(-1))^2 e^{-1}$

$= -1 + \frac{4}{e}$

$= \frac{4-e}{e}$

local max: $\left(-1, \frac{4-e}{e}\right)$

local min: $(1, -1)$

Section 4.5

#4) $y' = \frac{12x-3}{6x^2-3x+1}$

#8) $y' = 3(1+\ln x)^2 \cdot \frac{1}{x}$

#10) $y' = \frac{\ln x}{x}$

#12) $y' = \dfrac{(\ln 2x)(\frac{1}{x}) - (\ln x)(\frac{1}{\cancel{2}x}\cdot \cancel{2})}{(\ln 2x)^2}$

$= \dfrac{\ln 2x - \ln x}{x(\ln 2x)^2}$

$= \dfrac{\ln(\frac{2\cancel{x}}{\cancel{x}})}{x(\ln 2x)^2} = \dfrac{\ln 2}{x(\ln 2x)^2}$

#14) $y' = \dfrac{1}{1/x} \cdot \left(-\dfrac{1}{x^2}\right) = x\left(-\dfrac{1}{x^2}\right) = -\dfrac{1}{x}$

#16) $y' = \dfrac{-1}{(\ln x)^2} \dfrac{1}{x} = \dfrac{-1}{x(\ln x)^2}$

#20) $y' = \frac{1}{2}(\ln 2x)^{-1/2} \cdot \frac{1}{\cancel{2}x} \cdot \cancel{2}$

$= \dfrac{1}{2x\sqrt{\ln(2x)}}$

#29) $y = x^2 \ln x$

$y' = 2x\ln x + x^2 \cdot \frac{1}{x} = 2x\ln x + x$

$2x\ln x + x = 0$

$2\cancel{x}\ln x = -\cancel{x}$ ($x > 0$, so can divide both sides by $2x$

$\ln x = -\frac{1}{2}$

$x = e^{-1/2}$

$y(e^{-1/2}) = (e^{-1/2})^2 \ln(e^{-1/2}) = e^{-1}\left(-\frac{1}{2}\right) = \dfrac{-1}{2e}$

continued $\rightarrow$

So $\left(e^{-1/2}, -\frac{1}{2e}\right)$ is the extreme point

$$y'' = 2\ln x + 2x\cdot\frac{1}{x} + 1$$
$$= 2\ln x + 3$$
$$y''(e^{-1/2}) = 2\ln(e^{-1/2}) + 3$$
$$= 2\left(-\frac{1}{2}\right) + 3$$
$$= 2 > 0$$ so y is concave up.

$\Rightarrow$ $\boxed{\left(e^{-1/2}, -\frac{1}{2e}\right) \text{ is a local min.}}$

Section 4.6

#10) $e^{\ln x^2 + 3\ln y} = e^{\ln x^2} e^{3\ln y}$
$$= x^2 (e^{\ln y})^3$$
$$= x^2 y^3$$

#14) $\frac{1}{2}\ln 16 = \ln 16^{1/2} = \ln 4$

$\frac{1}{3}\ln 27 = \ln 27^{1/3} = \ln 3$

$4 > 3 \Rightarrow \ln 4 > \ln 3 \Rightarrow \frac{1}{2}\ln 16 > \frac{1}{3}\ln 27$

#16) a) $\ln 12 = \ln(4\cdot 3) = \ln 4 + \ln 3$

$= \ln(2^2) + \ln 3$

$= 2\ln 2 + \ln 3$

$\approx 2(.69) + 1.1 = 2.48$

b) $\ln 16 = \ln(2^4) = 4\ln 2 \approx 4(.69) = 2.76$

c) $\ln(9\cdot 2^4) = \ln 9 + \ln 2^4$

$= \ln(3^2) + \ln(2^4)$

$= 2\ln 3 + 4\ln 2 \approx 2(1.1) + 4(.69) = 4.96$

#23) $\ln x - \ln x^2 + \ln 3 = 0$

$\ln\left(\frac{3x}{x^2}\right) = 0$

$\ln\left(\frac{3}{x}\right) = 0$

$\frac{3}{x} = e^0 = 1$

$\boxed{3 = x}$

#24) $\ln\sqrt{x} - 2\ln 3 = 0$

$\ln x^{1/2} = 2\ln 3$

$\ln x^{1/2} = \ln(3^2)$

$x^{1/2} = 3^2$

$\boxed{x = 3^4 = 81}$

#38) $y = \ln\left[\dfrac{\sqrt{x}\,(x+1)^2(x+2)^3}{4x+1}\right]$

$= \ln\sqrt{x} + \ln(x+1)^2 + \ln(x+2)^3 - \ln(4x+1)$

$= \frac{1}{2}\ln x + 2\ln(x+1) + 3\ln(x+2) - \ln(4x+1)$

$y' = \frac{1}{2}\cdot\frac{1}{x} + 2\frac{1}{x+1} + 3\frac{1}{x+2} - \frac{4}{4x+1}$

$= \dfrac{1}{2x} + \dfrac{2}{x+1} + \dfrac{3}{x+2} - \dfrac{4}{4x+1}$