1. If a binary quadratic form has a square discriminant (i.e. factors) how can you describe which integers it represents? Try it for the following forms.

(a) \(x^2 - y^2\). (Hint: Odd numbers can be represented by \((y + 1)^2 - y^2\). Which even numbers can be represented?)

(b) \(x^2 - 10xy + 25y^2\).

(c) \(8x^2 + 22xy + 15y^2\). (Hint: factor it. If \(n\) is represented, then \(n = ab\). Which \(a, b\) are possible?)

2. Show that there are infinitely many proper representations of 1 by \(f(x, y) = x^2 - 2y^2\). (On the other hand, a positive definite (or a negative definite) form can represent a number \(n\) in only finitely many ways.)

(a) Find \(x_k, y_k\) such that \((3 + 2\sqrt{2})^k = x_k + \sqrt{2}y_k\). (Expand using the binomial formula.)

(b) Show that \((3 - 2\sqrt{2})^k = x_k - \sqrt{2}y_k\).

(c) Show that \(x_k^2 - 2y_k^2 = 1\).

(d) Show that \(x_{k+1} = 3x_k + 4y_k\) and \(y_{k+1} = 2x_k + 3y_k\) for \(k \geq 1\).

(e) Show that there are infinitely many proper representations of 1 by \(f\).

3. Find a reduced form equivalent to the form \(7x^2 + 25xy + 23y^2\).

4. Problem 3.5.5 from the book.

5. Let \(a, b, c\) be integers, \(p\) a prime.

(a) Show that \(ax^2 + by^2 + cz^2 \equiv 0 \pmod{p}\) has a non-trivial solution \(x, y, z\) (at least one of \(x, y, z \not\equiv 0 \pmod{p}\)).

(b) If \(p \geq 7\), show that if \(abc \not\equiv 0 \pmod{p}\) then \(ax^2 + by^2 + cz^2 \equiv 0 \pmod{p}\) has a solution with \(xyz \not\equiv 0 \pmod{p}\).

(c) Show that the equation \(r^2 + s^2 + 1 \equiv 0 \pmod{p}\) has a solution.

6. 3.6.12 from the book.