1. (a) Solve \( x^2 + 5x + 24 \equiv 0 \pmod{36} \)
(b) Solve \( x^3 + 10x^2 + x + 3 \equiv 0 \pmod{27} \)

2. Determine how many roots \( 9 + 3x + 3x^2 \) has mod \( 3^j \), for each \( j \geq 1 \).

3. Let \( p \) be a prime. Let \( S \) be the set of all polynomials \( g \) with \( \deg(g) \leq p - 1 \) and with coefficients in \( \{0, 1, \ldots, p - 1\} \)

(a) For \( n, m \geq 1 \) show that if \( n \equiv m \pmod{p - 1} \) then \( x^n \equiv x^m \pmod{p} \) for all \( x \).
(b) Use this fact to show that for every \( f(x) \in \mathbb{Z}[x] \) there is a \( g(x) \in S \) such that \( f(x) \equiv g(x) \pmod{p} \) for all \( x \).

4. For what values of \( n \) is \( \varphi(n) \) odd?

5. Show that there are infinitely many primes congruent to 3 mod 4. (Hint: if \( p_1, \ldots, p_n \equiv 3 \pmod{4} \) are primes show that \( N = p_1^2p_2^2 \cdots p_n^2 + 2 \) is divisible by some prime \( q \equiv 3 \pmod{4} \) such that \( q \neq p_1, \ldots, p_n \).)

6. Prove your answer, applying theorems such as Lagrange’s Theorem and Hensel’s Lemma when necessary.

(a) How many roots does \( x^2 - x \) have mod \( m \)? (You may have to phrase your answer in terms of the prime factorization of \( m \).)
(b) How many roots does \( x^{(p^2-p)(q-1)} - 1 \) have mod \( p^2q \) where \( p \) and \( q \) are distinct primes?