1. (a) Prove that $n(n + 1)$ is always even.
   (b) Prove that $6|n(n + 1)(n + 2)$.
   (c) Prove that $r!|n(n + 1)(n + 2) \cdots (n + r - 1)$. (Hint: think about binomial coefficients.)

2. Let $p$ be a prime number. Let $1 \leq r < p$.
   (a) Prove that $p$ is not a factor of $r!$ and thus that $p|\binom{p}{r}$.
   (b) Prove that $p|[(a + 1)^p - (a^p + 1)]$. (Hint: Expand $(a + 1)^p$ using the binomial theorem.)
   (c) Prove that if $p|(a^p - a)$ then $p|[(a + 1)^p - (a + 1)]$.
   (d) Prove that $p|(a^p - a)$ for all $a$.

3. Let $a, b \in \mathbb{Z}$ and $a \neq 0$. Prove that there exists integers $m, r$ such that $b = ma + r$ and $-(1/2)|a| \leq r < (1/2)|a|$.

4. Suppose $a = Ac + Bd$ and $b = Cc + Dd$.
   (a) Show that $(c, d)|(a, b)$.
   (b) If $AD - BC = \pm 1$, show that $(a, b) = (c, d)$. (Hint: Solve for $c, d$ in terms of $a, b$.)

5. Do parts a,b,c of 1.2.3, and do 1.2.24 and 1.2.25 of the book.

6. Prove $((n + 1)! + 1, n! + 1) = 1$ if $n \geq 1$. 