Minimizing Shortfall Risk Using Duality Approach - An Application to Partial Hedging in Incomplete Markets

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Option pricing and hedging in a complete market are well-studied with nice results using martingale theories. However, they remain as open questions in incomplete markets. In particular, when the underlying processes involve jumps, there could be infinitely many martingale measures which give an interval of no-arbitrage prices instead of a unique one. Consequently, there is no martingale representation theorem to produce a perfect hedge. The question of picking a particular price and executing a hedging strategy according to some reasonable criteria becomes a non-trivial issue and remains as an open question. Follmer and Leukert (2000) proposed an interesting partial-hedging strategy for European type options to reduce the initial capital charged while bearing some residual risk. The optimality criterion for measuring a hedging strategy is to minimize the shortfall risk at expiration. Unlike variance minimization, there is no penalty in case the hedging portfolio overshoots the option payoff. The existence of the optimal trading strategies when the stock price follows a semimartingale process is proved in Follmer and Leukert (2000) using the Neyman-Pearson lemma. However, computations of these strategies in incomplete models have turned out to be extremely difficult.

We are interested in doing some explicit computations in a jump diffusion setting which is widely studied as a typical incomplete market model. First we extend the duality results in Kramkov and Schachermayer (1999) to utility functions which are state dependent and not necessarily strictly concave in a semimartingale setting and apply them to the case of shortfall minimization.

This approach not only gives an alternative way of proving the existence of optimal solutions, but also gives a structural description of these solutions which is a generalization of the fact that in a complete market, the primal optimal terminal portfolio value is a constant times the marginal utility of the Radon-Nikodym derivative between the risk neutral measure and the physical measure. As a small exercise, we compute the optimal strategy, check the duality relationship and the HJB equation derived from dynamic programming principle in three complete market cases. Next we focus on the more interesting incomplete market case where we explicitly characterize the primal and dual sets in terms of the predictable characteristics. We provide upper bounds for the value function using duality results. For lower bounds, we pick a particular strategy which we call the 'bold strategy'. Closed-form solutions are given for the case of constant parameters in the jump diffusion model. In the cases of bonds and call options, the upper and lower bounds are quite tight for reasonable choice of parameters.