Optimal payoff profiles for law-invariant risk measures

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Abstract

Motivated by the problem of finding a risk-minimizing hedging strategy, we discuss a Neyman-Pearson type problem for law-invariant risk measures. Similar problems were studied by Huber and Strassen (1973), Cvitanic and Karatzas (2001), and others, mainly focussing on general existence results and the interpretation of solutions in terms of least favourable pairs of (probability) measures. Here, we use different methods and a more specific setting in order to obtain explicit solutions. Special emphasis is given to the class of quantile-based coherent risk measures that were introduced by Kusuoka (2001). We also discuss a class of risk measures that are derived from robust utility functionals. It turns out that replacing a classical utility funtional by a robust one leads to a diversification effect: There exists a critical target price $\widehat{v}$ below which the solution coincides with the classical one. Capital exceeding $\widehat{v}$, however, will be raised in a less risky or even risk-free manner.

References

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