

The No-Arbitrage Property Under A Change of Numéraire

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Computational Finance Seminar: April 8

Abstract

The fundamental idea in risk-neutral pricing is that asset prices discounted by money market accounts are martingales under a risk-neutral measure. However, under certain circumstances, other choices of numéraires are more useful. For example, to price bond options, we change from risk-neutral measure to forward measure and use bond prices as numéraires. To find a simple one-dimensional integro-differential equation for pricing arithmetic Asian options, the stock price is the proper numéraire to use. Of course, to build models of exchange rates, we will face two different currencies and therefore two different discounting factors as numéraires. A desirable property for the definition of no-arbitrage would be the stability under a change of numéraire. Unfortunately, this is not the case in the definition of No Free Lunch with Vanishing Risk (NFLVR) which is a version of no-arbitrage used by Delbaen and Schachermayer to prove the equivalence of no-arbitrage and existence of risk-neutral measure in semimartingale models. We will show an example when there is no-arbitrage under one currency and arbitrage under another. Then we will give a condition under which the no-arbitrage property is preserved under the change of numéraire. This talk is mainly based on the paper ‘The No-Arbitrage Property Under A Change of Numéraire’, by Delbaen and Schachermayer.