21-720 Fall 2009 R Pego

Measure and Integration

Outline of topics: This course treats deals with the Lebesgue integral in $\mathbf{R}^n$ in particular and the abstract theory of integration and measures in general. Topics covered include:

- Measurable sets, sigma-algebras. Measurable functions. Measures, measure spaces.
- Integrals and convergence theorems: monotone convergence, Fatou's lemma, Dominated convergence.
- Lebesgue measure, Borel measures, outer measure, Carathéodory's extension theorem
- Modes of convergence, theorems of Egoroff and Lusin
- Hahn and Jordan decompositions, Radon-Nikodym derivative. Lebesgue's differentiation theorem.
- Product measures, Fubini's theorem.
- L^p spaces, Riesz representation theorem. Hölder and Minkowski inequalities, completeness, equiintegrability, Vitali convergence theorem.

Prerequisites: Prospective students should note that the instructor will *not* regard 21-620 as a prerequisite for this course in Fall 2009.

Recommended text (available now via Amazon, e.g.):

Elements of Integration and Lebesgue Measure, by Robert G. Bartle, Wiley-Interscience, 1995. (Excellent for reading. Doesn't cover everything.)

Alternative main texts:

Measure Theory, by Paul R. Halmos, Springer-Verlag, 1974. (Classic.)

Real and Complex Analysis, by Walter Rudin, 3rd ed., McGraw-Hill, 1987. (A classic but very terse treatment, including a variety of other topics.)

Real analysis. Modern Techniques and Their Applications, by G. B. Folland, 2nd ed. Wiley-Interscience, 1999. (A useful collection of extra topics.)

Measure and Integral, by R. L. Wheeden and A. Zygmund, CRC Press, 1977.

Reference books:

Linear Operators, part I, by N. Dunford and J. Schwartz, Wiley-Interscience, 1988. (Chapter 3 is a classic reference on integration and measure theory for functions into Banach spaces.)

Measure Theory and Integration, by M. M. Rao, 2nd ed., M. Dekker, 2004.