

## **21-720 Measure and Integration Midterm Exam, Fall 2009**

The midterm exam is scheduled for **Tuesday, October 27 at 7pm** in the usual classroom (PPB 300). The test will be closed book, no notes or other aids. The exam will count somewhat more than one homework set toward the course grade: 80 points out of about 500 points for the course.

Test questions may involve statements of major theorems, proofs of results from class, portions of homework problems, or applications of concepts, results, and/or proof techniques developed in the course, related to the topics below. This material is based on the following material treated in class from Bartle: Chapters 11-14, and chapters 1-7, plus the Radon-Nikodým theorem.

### **Lebesgue measure**

Open cells in  $\mathbb{R}^p$ , Lebesgue outer measure. Carathéodory's condition, Lebesgue measurable sets, Lebesgue measure. Measurability of Borel sets.

### **Abstract measures and measurability**

$\sigma$ -algebras, measures. Measurable functions. Measurability of combinations and limits, limsup, liminf.

### **Integrals**

Simple functions, integrals, integrable functions. Monotone Convergence Theorem. Fatou's Lemma. Dominated Convergence Theorem. Integrals depending on a parameter. Definition of measures via integrals. Radon-Nikodým Theorem.

### **Lebesgue spaces**

Definition of  $L_p$  and  $L_\infty$  via equivalence classes. Norms, Hölder's and Minkowski's inequalities. Completeness.

### **Modes of convergence**

Almost everywhere convergence, convergence in measure, almost uniform convergence. Egoroff's theorem. Vitali's criteria for convergence in  $L_p$ .