Due Wednesday, Sept. 4:

1.1. Prove: any open subset of $\mathbb{R}^p$ is a countable union of half-open, pairwise disjoint cells of the form

$$I = \prod_{j=1}^{p} (a_j, b_j).$$

1.2. For each of the following, compute the Lebesgue outer measure:
(a) Any countable set.    (b) The standard Cantor set.  (c) $[0, 1] \setminus \mathbb{Q}$.

1.3. Suppose that $\mathcal{F}$ is a $\sigma$-algebra of subsets of $\mathbb{R}^p$ which contains every closed half-space of the form $\{x \in \mathbb{R}^p : x_j \geq c\}$, where $j \in \{1, \ldots, p\}$ and $c \in \mathbb{R}$ are fixed.
(i) Prove that $\mathcal{F}$ contains every open cell in $\mathbb{R}^p$ (i.e., every product of bounded open intervals).
(ii) Deduce that $\mathcal{F}$ contains every open set in $\mathbb{R}^p$.

1.4. Suppose $\mathcal{F}$ is a $\sigma$-algebra that has infinitely many elements.
(i) Show there is a countable family of pairwise disjoint sets in $\mathcal{F}$.
(ii) Show $\mathcal{F}$ is uncountable.

1.5. If $A \subset \mathbb{R}^2$ is a polygon, show that area $A = m^*(A)$.

(Reminder: Late homework will not be graded, per class policy.)