

21-640 Functional Analysis Midterm Exam, Spring 2007

The midterm exam is scheduled for Friday, March 30, 2:30-3:20, in the usual classroom. The test will be **open notes**, i.e., during the test you may consult written or printed notes that you bring, but not books or other sources. The exam will count somewhat more than one homework set toward the course grade: 80 points out of about 500 points for the course.

Test questions may involve proofs of results from class, portions of homework problems, or applications of concepts, results, and/or proof techniques developed in the course, related to the topics below:

Completion of a normed vector space. Operator norms, complete spaces of operators.

Hilbert space

Best approximation, orthogonal decomposition, Riesz representation theorem. Orthonormal bases, ℓ^2 , Bessel's inequality, Parseval's identity. Lax-Milgram theorem. Criteria for compactness in ℓ^2 .

Applications

L^2 convergence of Fourier series. Weak derivatives, periodic Sobolev spaces $H_{\text{per}}^s(\mathbb{R})$ and associated Sobolev inequalities. Weak solutions for a class of periodic differential equations on \mathbb{R} .

Banach space theory

Baire's theorem, Hahn-Banach theorem. Uniform boundedness principle, open mapping, inverse mapping, closed graph theorems. Spanning criterion. Duals, basic examples, double duals, quotients, sums, products

Weak topologies

Sequential weak convergence and sequential weak-* convergence. Weak and weak-* topologies. Convergence of nets. Compactness properties of the unit ball in X and X^* . Metrizability of the unit ball in separable X^* .