21-640 Functional Analysis Final Exam, Spring 2007

The final exam is scheduled for Friday, May 11, 5:30-8:30pm, in Porter Hall A18A. The test will be **open notes**, i.e., during the test you may consult written or printed notes that you bring, but not books or other sources. The exam will count about like two homework sets toward the course grade: 125 points out of about 500 points for the course.

Test questions may involve proofs of results from class, portions of homework problems, or applications of concepts, results, and/or proof techniques developed in the course, related to the topics below. The test is comprehensive, but will somewhat emphasize material in the second half of the course.

Completion of a normed vector space. Operator norms, complete spaces of operators.

Hilbert space

Best approximation, orthogonal decomposition, Riesz representation theorem. Orthonormal bases, ℓ^2, Bessel's inequality, Parseval's identity. Lax-Milgram theorem. Criteria for compactness in ℓ^2.

Applications

L^2 convergence of Fourier series. Weak derivatives, periodic Sobolev spaces $H^s_{\text{per}}(\mathbb{R})$ and associated Sobolev inequalities. Weak solutions for a class of periodic differential equations on $\mathbb{R}$.

Banach space theory

Baire's theorem, Hahn-Banach theorem. Uniform boundedness principle, open mapping, inverse mapping, closed graph theorems. Spanning criterion. Duals, basic examples, double duals, quotients, sums, products

Weak topologies

Sequential weak convergence and sequential weak-$*$ convergence. Weak and weak-$*$ topologies. Convergence of nets. Compactness properties of the unit ball in X and X^*. Metrizability of the unit ball in separable X^*.

Bounded operators

Norm, strong, weak operator topologies. Adjoints, relations between range and kernel. Hilbert adjoints. Self-adjoint operators, orthogonal projections.

Spectrum and resolvent of bounded operators

Point, continuous, residual spectrum. Neumann series. Spectrum and adjoints. Spectral radius.

Compact operators

Finite rank operators, norm limits, projections. Eigenspaces. Fredholm alternative. Discreteness of spectrum. Riesz projections. Spectral theorem for compact self-adjoint operators in Hilbert space.

Unbounded operators

Domains. Closed and closable operators. Resolvent. $a(x)d/dx$ on H^1_{per}.

OMIT: Riesz-Kakutani theorem, Potential theory, semigroup theory