Due Wednesday May 2:

• **P1:** (RS VI.20) Let X be a reflexive Banach space and let $T : X \to X$ be a bounded linear operator. Suppose that for every weakly convergent sequence $\{x_n\}$, the sequence $\{Tx_n\}$ converges in norm. Prove that T is compact.

• **P2:** (RS VI.43) Let M and N be subspaces of a Banach space X such that $M + N = X$ and $M \cap N = \{0\}$. Let P be the projection of X onto M. Prove that P is bounded if and only if both M and N are closed.

• **P3:** Suppose $\mathcal{H}$ is a separable Hilbert space, $A \in \mathcal{L}(\mathcal{H})$ is compact, and a sequence $\{B_n\}$ in $\mathcal{L}(\mathcal{H})$ converges strongly to B. Provide a counterexample showing that $\|AB_n - AB\| \to 0$ as $n \to \infty$ does not necessarily follow.

• **P4:** Let $\mathcal{H}$ be a Hilbert space, and $A \in \mathcal{L}(\mathcal{H})$. We say that a complex number λ lies in the *essential spectrum* of A if there exists an orthonormal sequence $\{u_n\}$ such that $\|(\lambda I - A)u_n\| \to 0$. Suppose that λ lies in the essential spectrum of A and that $B \in \mathcal{L}(\mathcal{H})$ is a compact operator. Show that λ lies in the essential spectrum of $A + B$.