21-630 Spring 2008 R Pego

Ordinary Differential Equations

Outline of topics

The course is a fundamental introduction to the qualitative theory of ordinary differential equations (ODE), particularly emphasizing *dynamical systems and geometric ideas*. We develop basic infrastructure for analyzing solutions of ODE (theorems on existence and uniqueness, dependence on parameters, continuation), study questions of stability and instability (using Liapunov functions, linearization, Floquet theory, dichotomies), develop geometric methods (Poincare-Bendixson index theory, attractors and flows, invariant manifold theorems) and study generic structure (normal forms, basic bifurcation theory). Much motivation comes from applications in many areas of science and engineering.

The initial value problem for systems of ODE: Existence for continuous systems. The Lipschitz condition. Uniqueness. Maximal interval of existence; continuation. Differentiability with respect to initial conditions.

Linear systems with constant coefficients: Exponential of a matrix. Jordan normal form; generalized eigenvectors. Log of a matrix. Criteria for stability, asymptotic stability.

General theory of linear systems of ODE: Linear independence, fundamental matrices. Abel's formula. Transport; evolution of phase space volume. Variation of constants formula. Nonlinear asymptotic stability of equilibria. Floquet theory: Floquet exponents, Floquet's theorem. Lyapunov exponents.

Invariant manifold theorem. Exponential matrix bounds. Necessary and sufficient integral equation. Contraction mapping principle. Local uniqueness of the stable manifold. Non-uniqueness of local center manifolds. Stability of the unstable manifold.

Omega-limit sets and attractors. Poincaré sections. Poincaré-Bendixson theory for 2D flows.

Implicit function theorem and applications: initial-value problems, stability and persistence of periodic orbits. Crandall-Rabinowitz bifurcation theorem. Normal forms. Fiber contraction theorem.