

21-236 Math Studies: Problem Seminar Batch 6

The groups should start working on problems according to (I,J,K,L)=(21,22,23,24), and continue freely as time allows.

21. Suppose we wish to partition the upper half square

$$S = \{(i, j) : 1 \leq i < j \leq n\}$$

into classes such that no class contains two points of the form $(i, j), (j, k)$. What is the minimum number of classes needed to do this?

22. Remove all squares from the sequence $1, 2, 3, \dots$. Show that the n -th term remaining has the form $n + m$ where m is the integer nearest to \sqrt{n} .
23. Suppose the points of the square

$$S = \{(i, j) : 1 \leq i, j \leq n\}$$

are labeled with the numbers $1, \dots, n^2$ (here distinct points get different labels so all labels are used). True or false: there are two adjacent points whose labels differ by at least n .

24. Each entry of an $n \times n$ matrix A is 0 or 1 independently with probability $\frac{1}{2}$. Over the field $\mathbb{Z}/2\mathbb{Z}$, what is the probability that $\det A = 1$?