

21-236 Math Studies: Problem Seminar Batch 5

The groups should start working on problems according to (I,J,K,L)=(17,18,19,20), and continue freely as time allows.

17. (a) Show that the number of non-congruent triangles with circumference $2n$ and integer sides is equal to the number of non-congruent triangles with circumference $2n - 3$ and integer sides.
(b) Show that these are also equal to the number of partitions of n into exactly three terms.
(c) Determine this number.

18. Prove that

$$\frac{\sin x}{x} = \prod_{i=1}^{\infty} \cos\left(\frac{x}{2^i}\right).$$

19. Recall the function $\mu(x, y)$ defined on a partially ordered set V from last time. Evaluate $\mu(x, y)$ if V is
 - (a) the lattice of all subsets of a set S ordered by inclusion
 - (b) the set of integers $1, \dots, n$, where $x \leq y$ means x divides y .
20. Suppose x and y are positive integers with $x > y\sqrt{2}$, and let

$$z = x + y\sqrt{2}.$$

We can then define two sequences of integers (x_n) and (y_n) by writing

$$z^n = x_n + y_n\sqrt{2}.$$

for $n = 1, 2, \dots$. Prove that

$$\lim_{n \rightarrow \infty} \frac{x_n}{y_n} = \sqrt{2}.$$