

## 21-235 Math Studies: Problem Seminar

12. Oddtown has  $n$  citizens and a collection of  $m$  clubs (each club is a subset of the citizens). Furthermore, the clubs have the property that each club has an odd number members and any two clubs from the collection have an even number of members in common. Prove  $m \leq n$ . (Hint: Consider the standard inner product on a vector space of the form  $F^n$ .)
13. Suppose that  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  maps the origin to the origin and preserves Euclidean distances. Show that  $f$  is linear:  $f(ax + by) = af(x) + bf(y)$  for all  $x, y \in \mathbb{R}^2$  and all  $a, b \in \mathbb{R}$ .