21-235 Math Studies: Problem Seminar

9. Suppose you have a finite collection of convex sets in the plane such that any three of them have some point in common. Show that they all have a point in common.

10. (From the UC Berkeley grad prelims) Consider $f : \mathbb{R}^2 \to \mathbb{R}$. Assume that for each fixed $x_0 \in \mathbb{R}$, $y \mapsto f(x_0, y)$ is continuous, and for each fixed $y_0 \in \mathbb{R}$, $x \mapsto f(x, y_0)$ is continuous. Find such an f that is not continuous.

11. Define a sequence $(s_n)_{n\in\mathbb{N}}$ recursively by setting $s_1 = 1$ and

$$s_{n+1} = \sqrt{s_1 + \ldots + s_n} \qquad \text{for } n \geq 1.$$

Show that

$$\lim_{n\to\infty} \frac{s_n}{n} = \frac{1}{2}.$$