

21-235 Math Studies: Problem Seminar

7. Suppose $(a_n)_{n \in \mathbb{N}}$ is a sequence of positive real numbers such that

$$a_{n+1} \leq \frac{a_n + a_{n-1}}{2}$$

for all $n \geq 2$. Prove that the sequence converges.

8. Let S be a finite collection closed intervals in the real line such that any two intervals in the collection have non-empty intersection. Show that there is a real number which belongs to every interval of the collection.