

21-235 Analysis Assignment 6

Problems due Friday December 7:

6.1. (Pugh p199 #68) (products of series)

6.2. (Pugh p253 #15) (modulus of continuity)

6.3. Let $[a, b] \subset \mathbb{R}$ and let $C^1 = C^1([a, b])$ denote the space of functions such that f is differentiable on $[a, b]$ and f' is continuous on $[a, b]$. On this vector space define

$$\|f\|_{C^1} = \|f\|_{\text{sup}} + \|f'\|_{\text{sup}}.$$

(a) Using the fundamental theorem of calculus, show that the map

$$f \mapsto Tf := \left(\int_a^b f(t) dt, f' \right)$$

is a continuous bijection from C^1 to the space $V = \mathbb{R} \times C^0$, equipped with norm

$$\|(c, g)\|_V = |c| + \|g\|_{\text{sup}} \quad \text{for } (c, g) \in V.$$

(b) If $f = T^{-1}(c, g)$, find a formula for $f(x)$, and show T is a homeomorphism.

In addition I suggest you try these problems from Pugh:

On pp. 186-200: 57, 59, 70 (try it!), 71

On pp. 251-257: 4a, 16a