21-235 Analysis Assignment 4

Problems due Friday Oct. 8

4.1. (a) Pugh p122 #68. (b) Pugh p133 #121.

4.2. Pugh p122 #64(a) Prove that every connected open subset of $\mathbb{R}^m$ is path-connected. (Hint: study the union of images of all paths starting from a given point.)

4.3. Prove that every open subset of $\mathbb{R}^m$ is a countable union of disjoint connected open sets. (This generalizes the structure theorem for open subsets of $\mathbb{R}$.)

4.4. Let X and Y be metric spaces with respective metrics d and ρ. Suppose X_1 and X_2 are subsets of X such that $X_1 \cup X_2 = X$. Suppose $f : X \to Y$ is such that the restrictions $f|_{X_1}$ and $f|_{X_2}$ are continuous. Which, if any, of the following statements are true and which may be false? (Give proofs or counterexamples as appropriate.)

a. The function f is automatically continuous.

b. If X_1 and X_2 are closed, then f is continuous.

c. If X_1 and X_2 are open, then f is continuous.

d. If X_1 is closed and X_2 is open, then f is continuous.

4.5. Pugh p49 #43(a) Remark: Convexity is necessary, but not sufficient by itself. To find a sufficient condition for a set S to be a unit ball for a norm, one should figure out how to use S to *define* a norm on $\mathbb{R}^2$, then prove the definition has the requisite properties.

In addition, it is worth thinking about these problems from Pugh, pp. 115-135: 34, 36, 54, 92, 122, 130.