

21-235 Analysis Assignment 4

Problems due Friday October 26:

4.1. (Pugh p119 #43) The *diameter* of a set S in a metric space is defined by

$$\text{diam } S = \sup\{d(p, q) : p, q \in S\}.$$

Suppose (K_n) is a nested sequence of compact non-empty subsets of a metric space M , with $K_{n+1} \subset K_n$ for all $n \in \mathbb{N}$, and $K = \bigcap_{n \in \mathbb{N}} K_n$. Suppose that for some $\mu > 0$, the diameter $\text{diam } K_n \geq \mu$ for all n . Is it true that $\text{diam } K \geq \mu$? Prove or give a counterexample.

4.2. (Pugh p119 #40) Let M be a metric space and $f : M \rightarrow \mathbb{R}$. The graph of f is the set

$$G = \{(p, y) \in M \times \mathbb{R} : y = f(p)\}.$$

- a. Prove that if f is continuous then its graph G is closed (as a subset of $M \times \mathbb{R}$).
- b. Prove that if f is continuous and M is compact then G is compact.
- c. Prove that if the graph G is compact then f is continuous.
- d. Give an example of a discontinuous $f : \mathbb{R} \rightarrow \mathbb{R}$ whose graph is closed.

4.3. (Pugh p119 #44) Suppose that M is a compact metric space and that \mathcal{U} is an open cover of M which is “redundant” in the sense that each $p \in M$ is contained in at least two members of \mathcal{U} . Show that \mathcal{U} has a finite subcover with the same property.

In addition, study these problems from Pugh, pp. 115-130: 38, 39, 42, 45–47.