

21-235 Analysis Assignment 3

Problems due Friday Sept. 17:

3.1. Prove or disprove: \mathbb{Q} is homeomorphic to \mathbb{N} .
(The metric is the usual $|x - y|$ for both.)

3.2. (a) Pugh p115 #11.
(b) Pugh p115 #5.

3.3. Let M be a metric space with metric d . The *distance* between a point $x \in M$ and a subset $T \subset M$ is defined as

$$\text{dist}(x, T) = \inf\{d(x, y) : y \in T\},$$

and the distance between two subsets S and T of M is defined as

$$\text{Dist}(S, T) = \inf\{d(x, y) : x \in S, y \in T\}.$$

(a) Prove that $\text{dist}(x, T) = 0$ if and only if x is a limit point of T .
(b) Prove or disprove: Dist necessarily satisfies the triangle inequality: for any three subsets S, T, R of M ,

$$\text{Dist}(S, T) \leq \text{Dist}(S, R) + \text{Dist}(R, T).$$

(c) Find subsets S and T of \mathbb{R} for which $S \cap T$ is empty with S and T nonempty, but

$$\sup_{x \in S} \text{dist}(x, T) = 0 = \sup_{y \in T} \text{dist}(y, S).$$

3.4. Show that every sequence of real numbers contains a monotone subsequence, i.e., one that is either non-increasing or non-decreasing.

In addition, study these problems from Pugh, pp. 115-130: 6, 18, 25, 26, 60.