

21-235 Analysis Assignment 3

Problems due Monday October 15:

3.1. Let X and Y be metric spaces with respective metrics d and ρ . Suppose X_1 and X_2 are subsets of X such that $X_1 \cup X_2 = X$. Suppose $f : X \rightarrow Y$ is such that the restrictions $f|_{X_1}$ and $f|_{X_2}$ are continuous. Which, if any, of the following statements are true and which may be false? (Give proofs or counterexamples as appropriate.)

- a. The function f is automatically continuous.
- b. If X_1 and X_2 are closed, then f is continuous.
- c. If X_1 and X_2 are open, then f is continuous.
- d. If X_1 is closed and X_2 is open, then f is continuous.

3.2. Let S be a subset of a metric space M . Prove:

- a. $\partial S = \partial S^c$.
- b. $\partial\partial S \subset \partial S$. Give an example to show that it is possible that $\partial\partial S \neq \partial S$.
- c. $\partial\partial S = \partial\partial\partial S$.
- d. If S is open, show $\text{cl}(\text{int}(\text{cl } S)) = \text{cl } S$. (Here cl denotes the closure of a set.)

3.3. Show that every sequence of real numbers contains a monotone subsequence, i.e., one that is either non-increasing or non-decreasing.

In addition, study these problems from Pugh, pp. 115-130: 34, 37, 84, 89, 91