21-132 Analysis II: Practice for Test 4 R. Pego

Test 4: 11:30 Wednesday, April 29, 2009

The material covered emphasizes chapter 11 from Apostol.

1. In each case below, say whether f_n converges pointwise on I, and whether f_n converges uniformly on I.

(a) $I = \mathbb{R}, \quad f_n(x) = \dfrac{1}{1+x^{2n}}.$

(b) $I = [2, \infty), \quad f_n(x) = \dfrac{1}{1+x^{2n}}.$

(c) $I = \mathbb{R}, \quad f_n(x) = n \text{ if } 0 < x < \dfrac{1}{n}, \quad f_n(x) = 0 \text{ otherwise.}$

2. In each case, say whether the series $\sum_{n=1}^{\infty} u_n(x)$ converges pointwise on I, and whether the series converges uniformly on I.

(a) $I = \mathbb{R}, \quad u_n(x) = \dfrac{(-1)^n x^{2n}}{(n!)^2}$

(b) $I = [-2, 2], \quad u_n(x) = \dfrac{(-1)^n x^{2n}}{(n!)^2}$

(c) $I = (0, 2), \quad u_n(x) = e^{-nx}$

3. For each complex power series below, find the radius of convergence

$$(a)\ \sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2 e^n} z^n \qquad (b)\ \sum_{n=0}^{\infty} n^5 (.2)^{n \log n} z^n \qquad (c)\ \sum_{n=0}^{\infty} \frac{z^n}{((2n)!)^{1/n}}$$

4. For each real power series below, determine the interval of convergence

$$I = \{x \in \mathbb{R} \mid \text{the series converges at } x\}.$$

Also, find an elementary function f whose value $f(x)$ equals the sum of the series:

$$(a)\ \sum_{n=2}^{\infty} \frac{n(n-1)(x+2)^n}{4^n} \qquad (b)\ \sum_{n=1}^{\infty} \frac{(x+2)^{n+1}}{n4^n}$$

5. Let $a \le b$ be given real numbers and suppose $\{f_n\}_{n=1}^{\infty}$ is a sequence of functions $f_n : [a, b] \to [0, \infty)$ such that

(i) for each $n \ge 1$, $x \mapsto f_n(x)$ is increasing in x,

(ii) for each $x \in [a, b]$, $n \mapsto f_n(x)$ is decreasing in n, and

(iii) $\lim_{n\to\infty} f_n(b) = 0$.

Prove $f_n \to 0$ uniformly on $[a, b]$.