21-132 Analysis II: Practice for Test 3 R. Pego
Test 3: 11:30 Friday, April 10, 2009

The material covered emphasizes chapter 10 from Apostol.

1. For each series below, determine whether it is divergent, conditionally convergent, or absolutely convergent. Justify your conclusions by citing the appropriate "tools" listed on the handout "Tools for establishing convergence/divergence for series." (This handout will be made available during the test.)

$$(a)\quad \sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}, \qquad (b)\quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\log n}{n}, \qquad (c)\quad \sum_{n=1}^{\infty} \frac{e^{-n}\sin n}{\sqrt{n}},$$

$$(d)\quad \sum_{n=1}^{\infty} \frac{n^n}{n!4^n}, \qquad (e)\quad \sum_{n=1}^{\infty} \left(2+\frac{1}{n}\right)^{-n}, \qquad (f)\quad \sum_{n=2}^{\infty} \log\left(\frac{n+1}{n-1}\right)$$

2. Let $x \in \mathbb{R}$ be arbitrary, and let

$$s_n = \sum_{k=1}^{n} \frac{\cos kx}{k^3}, \qquad S = \sum_{k=1}^{\infty} \frac{\cos kx}{k^3}.$$

Find N such that $|S - s_n| < 10^{-6}$ for all $n > N$.

3. In each part, test the improper integral for convergence.

$$(a)\quad \int_0^1 \sin\left(\frac{1}{x}\right) dx, \qquad (b)\quad \int_0^1 \frac{1}{\sqrt{1-x^2}}\, dx, \qquad (c)\quad \int_1^{\infty} \frac{x}{\cosh x}\, dx$$

4. (page 415, number 13) Given $a_n > 0$ for all $n \geq 1$. For each of the following statements, give a proof or exhibit a counterexample.

(a) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} a_n^2$ diverges.

(b) If $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.