

## 21-132 Assignment 8: due Tuesday March 31

(Problems in parentheses are recommended, but do not turn them in:)

**8.1-5.** From Apostol page 409, do problems 4, 7, 14, 35, 50, (1, 15, 16, 37, 40, 51, 52)

Hint for #4: To get started, use the comparison  $\frac{2n-1}{2n} = 1 - \frac{1}{2n} \leq e^{-1/2n}$ .

**8.6-8.** From Apostol page 420, do problems 3, 5, 19, (9, 13)

**8.9.** Let  $a_1 = 1$ ,  $a_2 = 2$ , and  $a_{n+1} = a_n + a_{n-1}$  for all  $n \geq 2$  (so  $a_n$  is the  $n$ -th Fibonacci number). Prove that

$$\sum_{n=1}^{\infty} \frac{1}{a_n}$$

converges. (Find a comparison to a geometric series.)

**8.10.** (a) Show that  $\int_1^{\infty} \frac{\sin x}{\sqrt{x}} dx$  converges.

(b) Show  $\int_1^{\infty} \frac{|\sin x|}{\sqrt{x}} dx$  diverges. (Maybe useful:  $\int_{k\pi}^{(k+1)\pi} |\sin x| dx = 1$ .)

(c) Show that  $\int_1^{\infty} \cos(t^2) dt$  converges.