

21-132 Assignment 7: due Tuesday March 24

(Problems in parentheses are recommended, but do not turn them in:)

7.1. From Apostol page 391, do problem 23

From Apostol page 393, do problems (4, 6, 7)

7.2-4. From Apostol page 398, do problems 8, 16, 19, (5, 7, 11)

7.5-8. From Apostol page 402, do problems 1, 6, 8, 14, (3, 4)

7.9. Using the result of problem 3.10 (on Assignment 3), prove that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}.$$

7.10. Show that for $s > 1$,

$$1 - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = (1 - 2^{1-s})\zeta(s)$$

where $\zeta(s)$ is the Riemann zeta function. Hint: the negative terms add up to

$$- \sum_{k=1}^{\infty} \frac{1}{(2k)^s}.$$