21-132 Assignment 5: due Tuesday February 24

Reading: Apostol, sections 9.7-9.9 (for Wednesday), 10.1-10.3 (For Friday and Monday).

The second TEST will be held in class Friday, February 27. It will cover chapter 7 (including Taylor polynomials and Taylor's formula with remainder, L'Hôpital's rule, limits at ∞ and infinite limits, limiting behaviors of $\log x$ and e^x), and chapter 9 (complex numbers, complex exponentials). and sections 10.1-10.3 (sequences of numbers).

(Problems in parentheses are recommended, but do not turn them in:)

5.1-3. From Apostol page 365, do problems (1bcg, 2acd, 3afgj,) 5acd, 7, 11ad.

5.4-6. From Apostol page 371, do problems (5, 7, 8, 10) 1ch, 2bc, 11

5.7-8. From Apostol page 382, do problems 14, 27 (5, 11, 13, 20, 25)

5.9. (Complex inversion maps circles to circles) Let $0 < r < a$, let z be a complex number, and let $w = 1/z$. Show that $|z - a| = r$ if and only if $|w - b| = s$, where

$$b = \frac{1}{2}\left(\frac{1}{a-r} + \frac{1}{a+r}\right) = \frac{a}{a^2 - r^2}, \quad s = \frac{1}{2}\left(\frac{1}{a-r} - \frac{1}{a+r}\right) = \frac{r}{a^2 - r^2}.$$

(Suggestion: Expand $(z-a)\overline{(z-a)} - r^2$ and divide by $z\bar{z}(a^2 - r^2)$.)

5.10. What is wrong with the following proof? (Something, I hope!)

Let $x = e^{\pi}$. Then
$x^i = -1$
$(x^i)^i = (-1)^i$
$x^{-1} = (-1)^i$
$(x^{-1})^i = (-1)^{i \cdot i} = -1$
$e^{-\pi i} = -1 = e^{\pi i}$
$(e^{-\pi i})^i = (e^{\pi i})^i$
$e^{\pi} = e^{-\pi}$
$\pi = -\pi$
$2\pi = 0$