21-132 Assignment 4: due Tuesday February 17

Reading: Apostol, sections 9.1-9.5 (For class Friday and Monday.)

(Problems in parentheses are recommended, but do not turn them in:)

4.1-2. From Apostol page 290, do problems (4, 12,) 16, 32

4.3-4. From Apostol page 295, do problems (10, 11) 3, 18

4.5-6. From Apostol page 303, do problems (1, 2, 9) 17, 27

4.7. (Compare: page 304 #31.) Let

$$f(x) = \begin{cases} e^{-1/x} & \text{for } x > 0, \\ 0 & \text{for } x \leq 0. \end{cases}$$

(a) Prove that for every integer $m \geq 0$, $f(x)/x^m \to 0$ as $x \to 0$.
(b) Prove that every integer $n > 0$, for $x > 0$ the n-th derivative

$$f^{(n)}(x) = e^{-1/x} P_n(1/x),$$

where $P_n(z)$ is a polynomial in z (of some degree).
(c) Prove that for every integer $n > 0$, $f^{(n)}(0)$ exists and equals 0.
(Note this means that all Taylor polynomials for f are zero!)

4.8. Let $a > 0$. Prove that

$$\lim_{x \to +\infty} \frac{\log x}{x^a} = 0$$

by the definition: I.e., prove $\forall \varepsilon > 0\ \exists N\ \forall x\ x > N \implies 0 < \dfrac{\log x}{x^a} < \varepsilon$.

4.9. Determine constants a, b and c such that

$$\log(\sinh x) = ax^2 + bx + c + o(1) \quad \text{as } x \to +\infty.$$

(At some point, using the Taylor polynomial approximation for $\log(1 - x)$ for x near 0 should come in handy.)