21-132 Assignment 3: due Tuesday February 3

The **first test** will be **Friday, February 6, 11:30am** in class. (The date is changed to allow a little more preparation time.) It will cover chapter 5 (fundamental theorem, substitution, integration by parts), chapter 6 (logs, exponentials, their polynomial approximations, hyperbolic and inverse trig functions, partial fractions) and sections 7.5-7.6 (Taylor's formula with bounds for the error).

Reading: Apostol, sections 7.1-7.6. (For class Friday and Monday.)

(Problems in parentheses are recommended, but do not turn them in:)

3.0. From Apostol page 251: (do problems 1, 5, 6, 23)

3.1-4. From Apostol page 256, do problems 23, 28, 29, 47 (2, 11, 30)

3.5-6. From Apostol page 267, do problems 5, 22

3.7-8. From Apostol page 284, do problems 2, 9

3.9. Find numbers a and b such that $f(x) = a\,\mathrm{sech}^2(bx)$ satisfies

$$f''(x) = f(x) - 3f(x)^2 \qquad \text{for all } x.$$

(The graph of f gives the shape of a *soliton* in the theory of waves.)

3.10. As on page 277, replacing x by $-x^2$ in (7.7) gives that for all x,

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - \ldots + (-1)^n x^{2n} + \frac{(-x^2)^{n+1}}{1+x^2}.$$

By integration, derive a formula for $E_{2n+1}(x)$ such that

$$\arctan x = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{2k+1} + E_{2n+1}(x)$$

for all x, and show that for all $x > 0$,

$$|E_{2n+1}(x)| \leq \frac{x^{2n+3}}{2n+3}.$$

(I suggest *not* to use Taylor's theorem and try to bound $f^{2n+2}(x)$ for $f(x) = \arctan(x)$. Instead, derive the formula for $E_{2n+1}(x)$ by integrating.)

Bonus(2pts): (Anti-approximation) If $x > 1$, can you prove a bound from below, $|E_{2n+1}(x)| \geq$ something that becomes large as n becomes large?