Reading: Apostol, chapter 6.
(Problems in parentheses are recommended, but do not turn them in:)

2.1-4. From Apostol page 236, do problems 3, 8, 22, 31 (2, 7, 15, 35)
2.5. From Apostol page 242, do problem 3.
2.6-8. From Apostol page 248, do problems 24, 39, 41 (11, 27, 35, 42)
2.9. (Differentiability of inverse functions) Suppose \( F : \mathbb{R} \to \mathbb{R} \) is a strictly increasing continuous function, and \( G \) is its inverse function: \( G(y) = x \) if and only if \( y = F(x) \). Suppose \( F \) is is differentiable at \( a \) with \( F'(a) > 0 \). Prove \( G \) is differentiable at \( b = F(a) \) with \( G'(b) = 1/F'(a) \).

Use an \( \epsilon \)-\( \delta \) argument, filling in the GAPs in the following argument.

Proof. We must show
\[
\lim_{y \to b} \frac{G(y) - G(b)}{y - b} = \frac{1}{F'(a)}.
\] (1)

Since \( F \) is differentiable at \( a \) and we know \( F'(a) \neq 0 \), we know from the basic limit theorems that
\[
\lim_{x \to a} \frac{x - a}{F(x) - F(a)} = \frac{1}{F'(a)}.
\] (2)

Let \( \epsilon > 0 \) be arbitrary. Then (2) means that there exists \( \delta_1 > 0 \) such that
\[
0 < |x - a| < \delta_1 \implies \left| \frac{x - a}{F(x) - F(a)} - \frac{1}{F'(a)} \right| < \epsilon.
\] (3)

Claim: There exists \( \delta_2 > 0 \) such that for all \( y \), \( 0 < |y - b| < \delta_2 \) implies \( 0 < |x - a| < \delta_1 \), where \( x = G(y) \) and \( a = G(b) \).

Proof of the claim: [GAP! You fill in the proof that \( \delta_2 \) exists.]

Using the claim, we finish the proof as follows: Suppose \( 0 < |y - b| < \delta_2 \). Let \( x = G(y) \), then \( 0 < |x - a| < \delta_1 \) and by (3) we have that
\[
\left| \frac{G(y) - G(b)}{y - b} - \frac{1}{F'(a)} \right| = \left| \text{[GAP!]} - \frac{1}{F'(a)} \right| < \epsilon.
\]

This proves (1). QED