

The material covered by test 3 emphasizes sections 2.6 through 3.8 from Apostol.

1. (a) Evaluate $\int_{-\pi/2}^{\pi} (\sin 2x - 2 \cos 3x) dx$

(b) Derive the identity $\cos 3x = 4 \cos^3 x - 3 \cos x$ and use it to evaluate

$$\int_0^b \cos^3 t dt.$$

2. Evaluate each of the following (and indicate your reasoning):

(a) $\lim_{x \rightarrow 0} \frac{\sqrt{1+3x} - 1}{x}$ (b) $\lim_{x \rightarrow 1^-} \frac{x^4 - 1}{|x^2 - 1|}$
(c) $\lim_{t \rightarrow 0} \frac{t \tan 5t}{\sin^2 3t}$ (d) $\lim_{x \rightarrow 0} \cos(x \sin(1/x))$

3. Use the ε - δ definition of limit to prove:

(a) $\lim_{x \rightarrow 0^+} \sqrt{x} \sin(1/x) = 0$.

(b) If f is L -continuous at p and $f(p) = 0$, then $\lim_{x \rightarrow p} \sin(f(x)) = 0$.

4. If $\lim_{x \rightarrow p} f(x) = p$, is it necessarily the case that $\lim_{x \rightarrow p} f(f(x)) = p$?
(Ans: No.) What additional condition on f will make it so?

5. Using $\sqrt{b} - \sqrt{a} = \int_a^b \frac{1}{2\sqrt{x}} dx$ and the fact $\sqrt{2025} = 45$, show that

$$45 - \frac{20}{89} < \sqrt{2005} < 45 - \frac{20}{90}.$$

Problems from Apostol: page 114 #11; page 120 #20

Problems from Apostol: page 138 #24, 28; page 142 #16