21-131 Analysis I: Practice for Test 1 R. Pego
Test 1: 11:30 Friday September 19, 2008

For all problems, solutions must explain how you reach your conclusions.

The material covered by test 1 is Apostol chapter I.

1. Suppose P and Q are logical statements (true or false). Use truth tables to determine which of the following are equivalent.

(i) $P \wedge Q$

(ii) $\neg(P \to Q)$

(iii) $\neg(P \to \neg Q)$

2. Suppose A, B and C are sets, and suppose $A \cap B \subset C$. Show that if $a \in A$, then $a \notin B \setminus C$.

3. Suppose a, b, c, d are real numbers and suppose $0 < a < b$ and $d > 0$. Prove that if $ac \geq bd$ then $c > d$.

4. Suppose $x > -1$. Prove by mathematical induction that for all $n \in \mathbb{N}$,

$$(1 + x)^n \geq 1 + nx.$$

5. Let S be a non-empty set of real numbers that has an upper bound. Define

$$T = \{2x + 1 \mid x \in S\}.$$

(i) Prove that $2\sup(S) + 1$ is an upper bound for T.

(ii) Prove that $2\sup(S) + 1$ is the least upper bound for T.

6. Let A and B be two non-empty bounded sets of real numbers.

(i) Show that if $\sup A < \inf B$ then A and B are disjoint sets.

(ii) Suppose that $\inf A < \sup B$. Show that there exist $a \in A$ and $b \in B$ such that $a < b$.

7. Let $x \geq 1$. Show that there exists a positive integer n such that

$$n^3 \leq x < (n + 1)^3.$$