For all problems, solutions must explain how you reach your conclusions.

The material covered by test 1 is Apostol chapter I.

1. Suppose $P$ and $Q$ are logical statements (true or false). Use truth tables to determine which of the following are equivalent.
   
   (i) $P \land Q$
   
   (ii) $\neg (P \rightarrow Q)$
   
   (iii) $\neg (P \rightarrow \neg Q)$

2. Suppose $A$, $B$ and $C$ are sets, and suppose $A \cap B \subset C$. Show that if $a \in A$, then $a \notin B \setminus C$.

3. Suppose $a, b, c, d$ are real numbers and suppose $0 < a < b$ and $d > 0$. Prove that if $ac \geq bd$ then $c > d$.

4. Suppose $x > -1$. Prove by mathematical induction that for all $n \in \mathbb{N}$,
   
   $$(1 + x)^n \geq 1 + nx.$$ 

5. Let $S$ be a non-empty set of real numbers that has an upper bound. Define
   
   $$T = \{2x + 1 \mid x \in S\}.$$ 

   (i) Prove that $2\sup(S) + 1$ is an upper bound for $T$.
   
   (ii) Prove that $2\sup(S) + 1$ is the least upper bound for $T$.

6. Let $A$ and $B$ be two non-empty bounded sets of real numbers.

   (i) Show that if $\sup A < \inf B$ then $A$ and $B$ are disjoint sets.

   (ii) Suppose that $\inf A < \sup B$. Show that there exist $a \in A$ and $b \in B$ such that $a < b$.

7. Let $x \geq 1$. Show that there exists a positive integer $n$ such that
   
   $$n^3 \leq x < (n+1)^3.$$