21-131 Assignment 9: due Tuesday October 28

9.1–2. From Apostol page 119, do problems 8, 15a

9.3–4. From Apostol page 124, do problem 19

9.5. Suppose f is decreasing on $[a, b]$. Let $x, y \in [a, b]$ and assume $y < x$. Let $z \in (y, x)$.

(i) Compare $f(z)$ with the average value of f on $[y, z]$. Which is larger and why?

(ii) Compare $f(z)$ with the average value of f on $[z, x]$. Which is larger and why?

(iii) Let F be an indefinite integral of f. Using the answers to (i) and (ii), show that F is concave on $[a, b]$. I.e., show that for all $x, y, z \in [a, b]$, if $y < z < x$ then

$$\frac{F(z) - F(y)}{z - y} \geq \frac{F(x) - F(z)}{x - z}.$$

Interpret this inequality geometrically in terms of the graph of F.

9.6. Recall we say that a function f on $I \subseteq \mathbb{R}$ is L-continuous if and only if

$$|f(x) - f(y)| \leq L|x - y| \quad \forall x, y \in I.$$

Suppose f is L-continuous on $[a, b]$ for some $L > 0$. Show that for all positive integers n there exists a step function s_n with the property that

$$|f(x) - s_n(x)| \leq \frac{C}{n} \quad \text{for all } x \in [a, b],$$

where $C = L(b - a)$.

9.7–10. From Apostol page 138, do problems 3, 12, 17, 20