21-131 Assignment 7: due Tuesday October 14

7.1–4. From Apostol page 105, do problems 16, 23, 26, 30.

7.5. From Apostol page 111, do problems 5, 11.

Extra problems, not required to be turned in:

7.E1. Suppose $0 < y < z$. Using induction, prove that for every integer $p \geq 1$,

$$py^{p-1}(z-y) \leq z^p - y^p \leq pz^{p-1}(z-y).$$

7.E2. Using the result in E1 together with the identity (telescoping sum)

$$\frac{1}{x_0^p} - \frac{1}{x_n^p} = \sum_{k=1}^{n} \left(\left(\frac{1}{x_{k-1}} \right)^p - \left(\frac{1}{x_k} \right)^p \right),$$

show that if $0 < a < b$ and p is a positive integer, then

$$p \int_a^b \frac{1}{x^{p+1}}\, dx = \frac{1}{a^p} - \frac{1}{b^p}$$

(Suggestion: Use theorem 1.14 from Apostol.)

7.E3. Try to prove that for all x and each positive integer n,

$$\sin nx = 2^{n-1} \prod_{k=0}^{n-1} \sin \left(x + \frac{k\pi}{n} \right).$$

(I came across this in a book of identities, without a proof. If you find a proof I would be very interested to see it!)