

21-131 Assignment 4: due Tuesday September 23

4.1. (cf. Apostol page 60 problem 4) A point (x, y) in the plane is called a *lattice point* if both x and y are integers. Let P be a *lattice polygon*, meaning a polygon whose vertices are lattice points, and let

$$\alpha(P) = I + \frac{1}{2}B - 1,$$

where I is the number of lattice points in the interior of P and B is the number of lattice points on the boundary of P .

1. Show that if P_1 and P_2 are lattice polygons that share an edge E , then $\alpha(P) = \alpha(P_1) + \alpha(P_2)$.
2. Show that if P is a rectangle with sides parallel to the coordinate axes, then $\alpha(P)$ is the area of P .
3. Show that if P is a triangle with two sides parallel to the coordinate axes, then $\alpha(P)$ is the area of P .
4. Show that $\alpha(P)$ is the area of P for any lattice triangle (polygon with three edges).
5. Use induction on the number of edges to prove $\alpha(P)$ is the area of P for any lattice polygon.

4.2. From Apostol, page 70, problem 3: Show that

$$\int_a^b [x] dx + \int_a^b [-x] dx = a - b.$$

4.3. From Apostol, page 72, problem 1.7: Suppose that $a < b$ and that s is a step function. Prove that for any real c ,

$$\int_a^b s(x) dx = \int_{a+c}^{b+c} s(x-c) dx.$$

(See the proof of Theorem 1.8 on p. 71.)

Recommended: From Apostol page 70, do problem 1 (and more as time allows) and check the answers in the back of the book.