

21-131 Assignment 12: due Tuesday November 25

(Problems in parentheses are recommended exercises, not to turn in.)

12.1–2. From Apostol page 167, do problems 24, 38 (6, 7, 11, 12, 14, 36)

12.3. From Apostol page 173, do problems 11 (15, 16)

12.4–7. From Apostol page 179, do problems 7, 14, 23, 32 (8, 9, 17)

12.8–9. From Apostol page 186, do problems 4, 8

12.10. Suppose $F(x) = xf(x)$. Suppose f is n times differentiable in (a, b) . Guess a formula for the n th derivative $F^{(n)}(x)$ in (a, b) and prove it using induction (LCR).

12.E1. Bonus problem: Prove the general Leibniz rule: For every integer $n \geq 1$, if functions f and g are n times differentiable on (a, b) and $F(x) = f(x)g(x)$ on (a, b) then

$$F^{(n)}(x) = \sum_{k=0}^n \binom{n}{k} f^{(k)}(x)g^{(n-k)}(x), \quad \text{where} \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$