21-131 Assignment 10: due Tuesday November 11

10.1–3. From Apostol page 139, do problems 27, 29, 33bc

10.4–6. From Apostol page 142, do problems 17, 19, 23

10.7. Suppose f_1 and f_2 are functions defined on $\mathbb{R}$, and suppose that f_1 is L_1-continuous and f_2 is L_2-continuous, where L_1, $L_2 \geq 0$. Prove that $f_2 \circ f_1$ is L-continuous with $L = L_1 L_2$.

10.8. A point p is called a *fixed point* of a function f if and only if $p = f(p)$. Suppose that f is L-continuous on $\mathbb{R}$ for some number $L \in [0, 1)$. Show that f cannot have two different fixed points.

10.E1. Bonus problem: Consider the "$\mathbb{Q}$-staircase function" from Assignment 4:

$$f(x) = \sup_n \sum_{k=1}^{n} \left(\frac{1}{2}\right)^k H(x - r_k),$$

where r_1, r_2,... is a list of all rational numbers. Show that f is continuous at p for all *irrational* numbers p.